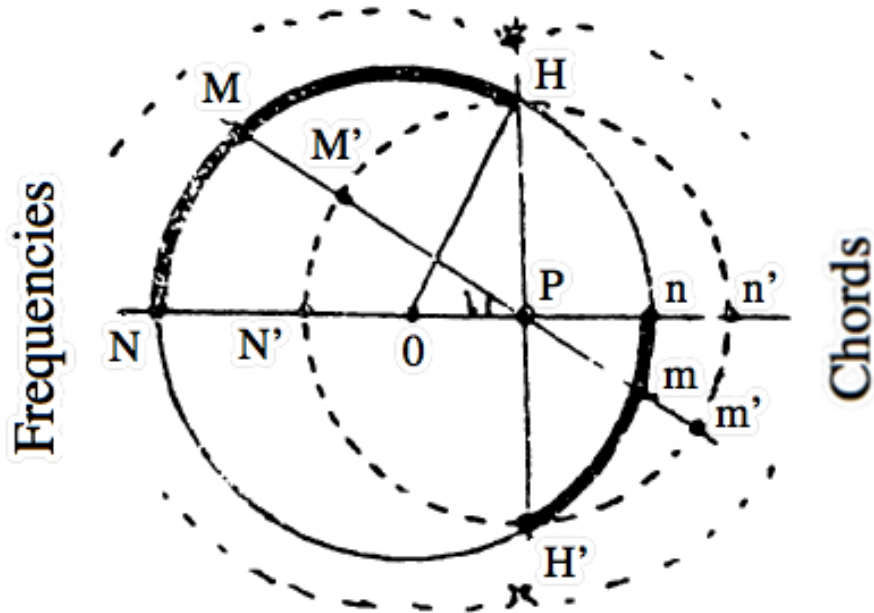


On p. 97 of *La leçon de Platon*, by Dom Néroman (La Bégude de Mazenc, Arma Artis, 2002), we see a figure that I am copying below, with my translations substituted for the original words in French:



The author examines the case in which

$$PH = 4,$$

$$OP = \frac{3}{5}OH = \frac{3}{4}PH,$$

$$PH = \frac{4}{5}OH,$$

$$OH = \frac{5}{4}PH.$$

Then he says that the ratio $\frac{PM}{PH} = \frac{PH}{4\sqrt{1+(\tan i)^2}} \left(3 + \sqrt{25 + 16(\tan i)^2} \right)$. He does not demonstrate that, I have tried to do it myself, but I have not succeeded so far. I have tried making use of the law of cosines to determine PM, knowing PH (4, in our case), OP = 3, and OH = OM = 5. Then, if $OM^2 = OP^2 + PM^2 - 2OP \cdot PM \cdot \cos i$, in our case, $25 = 9 + PM^2 - 6 \cdot PM \cdot \cos i \leftrightarrow PM^2 - 6 \cdot PM \cdot \cos i - 16 = 0 \leftrightarrow PM =$

$$\frac{(6 \cdot \cos i) \pm \sqrt{36(\cos i)^2 + 64}}{2}.$$

To put this as a function of $\tan i$, we may remember that $\cos i = \frac{1}{\sqrt{1+(\tan i)^2}}$. Then:

$$\begin{aligned}
PM &= \frac{(6 \cdot \cos i) \pm \sqrt{36(\cos i)^2 + 64}}{2} = \frac{\left(\frac{6}{\sqrt{1 + (\tan i)^2}}\right) \pm \sqrt{\frac{36}{1 + (\tan i)^2} + 64}}{2} \\
&= \frac{\left(\frac{6}{\sqrt{1 + (\tan i)^2}}\right) \pm \sqrt{\frac{36 + 64(1 + (\tan i)^2)}{1 + (\tan i)^2}}}{2} \\
&= \frac{\left(\frac{6}{\sqrt{1 + (\tan i)^2}}\right) \pm \frac{\sqrt{36 + 64(1 + [\tan i]^2)}}{\sqrt{1 + (\tan i)^2}}}{2} \\
&= \frac{6 \pm \sqrt{36 + 64(1 + [\tan i]^2)}}{2\sqrt{1 + (\tan i)^2}},
\end{aligned}$$

and now we may write $\frac{PM}{PH} = \frac{\frac{6 \pm \sqrt{36 + 64(1 + [\tan i]^2)}}{2\sqrt{1 + (\tan i)^2}}}{4} = \frac{6 \pm \sqrt{36 + 64(1 + [\tan i]^2)}}{8\sqrt{1 + (\tan i)^2}}$, but that is not equal to what Néroman says, namely $\frac{h}{4\sqrt{1 + (\tan i)^2}} \left(3 + \sqrt{25 + 16(\tan i)^2}\right)$. Any help to identify and fix my mistake (or Néroman's?) will be welcome.