



Damping

Spring $\Rightarrow C_s \dot{a}$ and $C_s \dot{b}$

Pendulum $\Rightarrow C_p \dot{\theta}_1$ & $C_p \dot{\theta}_2$

$$T = \frac{1}{2} m_1 \dot{a}^2 + \frac{1}{2} m_2 \dot{b}^2 + \frac{1}{2} m_1 (r+a)^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 (r+b)^2 \dot{\theta}_2^2$$

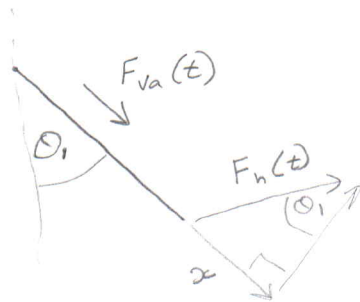
$$V = \frac{1}{2} k a^2 + \frac{1}{2} k b^2 - m_1 g (r+a) \cos \theta_1 - m_2 g (r+b) \cos \theta_2$$

a. coordinate

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{a}} \right) = m_1 \ddot{a} \quad \frac{\partial \mathcal{L}}{\partial a} = \frac{m_1 (2a+2r) \dot{\theta}_1^2}{2} - ka + gm_1 \cos \theta_1$$

$$\frac{\partial \mathcal{D}}{\partial \dot{a}} = C_s \dot{a} \quad m_1 \dot{\theta}_1^2 (a+r)$$

$$m_1 \ddot{a} = \frac{m_1 (2a+2r) \dot{\theta}_1^2}{2} + ka - gm_1 \cos \theta_1 + C_s \dot{a} = Q_a$$



$$\sin \theta_1 = \frac{x}{F_h(t)}$$

$$x = F_h(t) \sin \theta_1$$

$$(\delta_1 + \delta_2) Q_x = F_h(t) [\sin \theta_1 \hat{i} + \cos \theta_1 \hat{j}] \cdot \delta_1 + F_v(t) \delta_2 \uparrow \left\{ \begin{array}{l} \text{Similarly} \\ \text{for } b \\ \text{only } 16.67 \text{ Hz} \\ \text{for } b \end{array} \right.$$

$$\Rightarrow F_h(t) \sin \theta_1 + \underbrace{F_v(t)}_{\text{occurs twice}}$$

occurs twice
@ parametric frequency
16.67 Hz

$$m_1 \ddot{a} = Q_a - m_1 \dot{\theta}_1^2 (a+r) + ka - gm_1 \cos \theta_1 - C_s \dot{a}$$

$$\ddot{a} = \frac{Q_a + m_1 \dot{\theta}_1^2 (a+r) - ka + gm_1 \cos \theta_1 - C_s \dot{a}}{m_1}$$

b coordinate

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{b}} \right) = m_2 \ddot{b} \quad \frac{\partial \mathcal{L}}{\partial b} = \frac{m_2 (2b + 2r) \dot{\theta}_2^2}{2} - kb + gm_2 \cos \theta_2$$

$$\frac{\partial D}{\partial \dot{b}} = C_s \dot{b}$$

$$m_2 \dot{\theta}_2^2 (r+b)$$

$$m_2 \ddot{b} + \frac{m_2 (2b + 2r) \dot{\theta}_2^2}{2} + kb - gm_2 \cos \theta_2 + C_s \dot{b} = Q_b$$

$$m_2 \ddot{b} = Q_b - m_2 \dot{\theta}_2^2 (r+b) + kb - gm_2 \cos \theta_2 - C_s \dot{b}$$

$$\ddot{b} = \frac{Q_b + m_2 \dot{\theta}_2^2 (r+b) - kb + gm_2 \cos \theta_2 - C_s \dot{b}}{m_2}$$

θ_1 coordinate

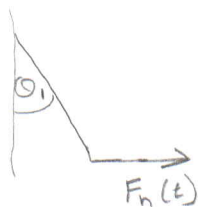
$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} = m_1 \dot{\theta}_1 (r+a)^2$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} \right) = m_1 \ddot{\theta}_1 (a+r)^2 + 2m_1 \dot{\theta}_1 \dot{a} (r+a)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = -gm_1 \sin \theta_1 (a+r)$$

$$\frac{\partial \mathcal{D}}{\partial \dot{\theta}_1} = C_p \dot{\theta}_1$$

$$m_1 \ddot{\theta}_1 (r+a)^2 + 2m_1 \dot{\theta}_1 \dot{a} (r+a) + gm_1 \sin \theta_1 (a+r) + C_p \dot{\theta}_1 = Q_{\theta_1}$$



$$\begin{aligned} \delta Q_1 Q_{\theta_1} &= F_h(t) [\sin \theta_1 \hat{i} + \cos \theta_1 \hat{j}] (a+r) \delta \theta_1 \\ &\Rightarrow (a+r) F_h(t) \cos \theta_1 \end{aligned} \quad \left. \begin{array}{l} \text{Similarly for} \\ \theta_2 \end{array} \right\}$$

$$m_1 \ddot{\theta}_1 (r+a)^2 = Q_{\theta_1} - 2m_1 \dot{\theta}_1 \dot{a} (r+a) + gm_1 \sin \theta_1 (a+r) - C_p \dot{\theta}_1$$

$$\ddot{\theta}_1 = \frac{Q_{\theta_1} - 2m_1 \dot{\theta}_1 \dot{a} (r+a) - gm_1 \sin \theta_1 (a+r) - C_p \dot{\theta}_1}{m_1 (r+a)^2}$$

θ_2 coordinate

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = m_2 \dot{\theta}_2 (r+b)^2 \quad \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} \right) = m_2 \ddot{\theta}_2 (r+b)^2 + 2 m_2 \dot{\theta}_2 \dot{b} (r+b)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_2} = -g m_2 \sin \theta_2 (r+b)$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2} = C_p \dot{\theta}_2$$

$$m_2 \ddot{\theta}_2 (r+b)^2 + 2 m_2 \dot{\theta}_2 \dot{b} (r+b) + g m_2 \sin \theta_2 (r+b) - C_p \dot{\theta}_2 = Q_{\theta_2}$$

$$m_2 \ddot{\theta}_2 (r+b)^2 = Q_{\theta_2} - 2 m_2 \dot{\theta}_2 \dot{b} (r+b) + g m_2 \sin \theta_2 (r+b) - C_p \dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{Q_{\theta_2} - 2 m_2 \dot{\theta}_2 \dot{b} (r+b) + g m_2 \sin \theta_2 (r+b) - C_p \dot{\theta}_2}{m_2 (r+b)^2}$$

where $(b+r)$ & \dot{b} are replaced with the
holonomic constraint