https://www.physicsforums.com/threads/invariance-of-energy-momentum-relativistic.999689/

$$(\frac{E}{c})^2 - p^2 \; is \; invariant$$

Exactly:

$$(\frac{E}{c})^2 - p^2 = (mc)^2 \qquad (1)$$

As p is equal to $\frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$, it follows

$$\left(\frac{E}{c}\right)^2 - \frac{m^2v^2}{1-\left(\frac{v}{c}\right)^2} = (mc)^2 \Rightarrow \; E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad (2)$$

And for the observer O '

$$E' = \frac{mc^2}{\sqrt{1-\left(\frac{v'}{c}\right)^2}} \qquad (3)$$

Substituting v' in terms of v and u, that is

$$v' = \frac{v-u}{1-\frac{vu}{c^2}}$$

$$v'^2 = \frac{v^2+u^2-2vu}{\left(1-\frac{vu}{c^2}\right)^2} \qquad (4)$$

$$1-\left(\frac{v'}{c}\right)^2 = 1-\frac{v^2+u^2-2vu}{c^2\left(1-\frac{vu}{c^2}\right)^2} = \frac{c^2+\frac{v^2u^2}{c^2}-2vu-v^2-u^2+2vu}{c^2\left(1-\frac{vu}{c^2}\right)^2}$$

$$= \frac{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}}{\left(1-\frac{uv}{c^2}\right)^2} \qquad (5)$$

Substituting (5) in (3)

$$E' = \frac{mc^2}{\sqrt{1-\left(\frac{v'}{c}\right)^2}} = \frac{mc^2\left(1-\frac{vu}{c^2}\right)}{\sqrt{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}}} \qquad (6)$$

(6) is the (a) solution.

Let's see part b): Reduce the expression

$$\left(\frac{E'}{c}\right)^2 - \frac{m^2 v'^2}{1-\left(\frac{v'}{c}\right)^2} \qquad (7)$$

From (1) we know that (7) is equal to $(mc)^2$. Let's see it.

Substituting (6), (5) and (4) in (7):

$$\frac{mc\left(1-\frac{vu}{c^2}\right)^2}{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}} - \frac{m^2\frac{v^2+u^2-2vu}{\left(1-\frac{vu}{c^2}\right)^2}}{\frac{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}}{\left(1-\frac{uv}{c^2}\right)^2}} = \frac{m^2c^2\left(1-\frac{vu}{c^2}\right)^2}{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}} - \frac{m^2(v^2+u^2-2vu)}{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}}$$

$$= \frac{m^2c^2\left(1+\frac{v^2u^2}{c^4}-\frac{2vu}{c^2}\right)-m^2(v^2+u^2-2vu)}{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}}$$

$$= \frac{m^2c^2\left(1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}\right)}{1+\frac{v^2u^2}{c^4}-\frac{u^2+v^2}{c^2}} = m^2c^2$$

This is, the simplest way of $E'^2-(p'c)^2=(mc^2)^2$