

If the magnetic field  $B$  in the frame where the current carries a flowing charges equals to:

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnetic field in the frame where the charges are at rest should equal to:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$

It does not change in sign because it is positive charge in the first frame. In the second frame it the charge are at rest but the negative charge are moving in the opposite direction and the observer still considers a positive current moving in the same direction as the first observer.

Also  $r=r'$

So  $\mathbf{B} = \mathbf{B}'$  (1)

Now going to the transformation equation in the link:

$$\mathbf{B}'_{\perp} = \gamma (\mathbf{B}_{\perp} - \mathbf{v} \times \mathbf{E}/c^2),$$

First I am wondering that  $E=0$  because the moving positive charge in one direction is balanced by moving negative charge in the opposite direction.

Then:

$$\mathbf{B}'_{\perp} = \gamma (\mathbf{B}_{\perp}) \quad (2)$$

From 1 and 2 ,  $B$  cannot be equal to  $B'$  except if  $v=0$ .