Homework 1 Solution

CBE 60553

September 2, 2014

Problem 1

A→B

$$U = 2.5PV + C, \qquad U_{AB} = U_B - U_A = 2.5[(PV)_B - (PV)_A] = 10000\ J$$

$$\boxed{W_{AB} = -P\int_{V_A}^{V_B} dV = -4000\ J}$$

$$U_{AB} = Q_{AB} + W_{AB}$$

$$\boxed{Q_{AB} = 14000\ J}$$

B→C

Need pressure as a function of volume along this path. From the figure, the relationship is linear and given by

$$P(V) = -15 \times 10^6 V + 0.65 \times 10^6$$

Integrate to find the work

$$\boxed{W_{BC} = -\int_{V_B}^{V_C} PdV = -\left[\frac{-15 \times 10^6 V^2}{2} + 0.65 \times 10^6 V\right]_{V_B}^{V_C} = 7000\ J}$$

From our expression for U

$$U_{BC} = -2500\ J$$

$$\boxed{Q_{BC} = -9500\ J}$$

C→A

$$U_{CA} = U_C - U_A = -7500\ J$$

Since volume is constant

$$\boxed{W_{CA} = 0}$$

Which means

$$\boxed{Q_{CA} = -7500\ J}$$

Total change A→B→C→A

$$Q_{cycle} = 14000 - 7500 - 9500 = -3000\ J$$
$$W_{cycle} = -4000 + 0 + 7000 = 3000\ J$$
$$U_{cycle} = Q_{cycle} + W_{cycle} = 0\ J$$

Problem 2

Along the Parabola

$$P = 10^5 + 10^9 \times (V - 0.02)^2$$

the work can be found by integration

$$\boxed{W_{AB} = -\int_{V_A}^{V_B} PdV = \left[-500000V + 2\times 10^7 V^2 - 3.333\times 10^8 V^3\right]_{.01}^{.03} = -2666.67\ J}$$

Since

$$U_{AB} = 10000$$

then

$$\boxed{Q_{AB} = 12666.67\ J}$$

Problem 3

For an adiabat we know that

$$dU = -PdV$$

and we can also write

$$dU = \left.\frac{\partial U}{\partial V}\right|_P dV + \left.\frac{\partial U}{\partial P}\right|_V dP$$

and we have the expression for U

$$U = 2.5PV + C$$

taking the partial derivatives

$$dU = 2.5PdV + 2.5VdP = -PdV, \qquad -3.5PdV = 2.5VdP$$
$$5VdP = -7PdV, \qquad -5\int_1^P \frac{dP}{P} = 7\int_1^V \frac{dV}{V}$$
$$-5\ln P = 7\ln V + C, \qquad 5\ln P + 7\ln V = C$$
$$\boxed{P^5V^7 = C}$$

Problem 4

There are three postulates we are testing for

II

$$\frac{\partial S}{\partial U} > 0$$

III

$$S(\lambda U, \lambda V, \lambda N) = \lambda S(U, V, N)$$

IV

$$\frac{\partial U}{\partial S} = 0, \text{ as } S \to 0$$

Use Mathematica script to test all three postulates for each function given
For example, for equation a,

$$\begin{aligned}
&\text{Sa}[\text{u}_, \text{v}_, \text{n}_] := \left(R^2 / (v * \theta)\right)^{1/3} * (n * v * u)^{1/3} \\
&(\text{*II*})\text{Sa}[\lambda, \lambda, \lambda] \\
&(\text{*III*})D[\text{Sa}[U, 1, 1], U] \\
&(\text{*IV*})D[\text{Solve}[\text{Sa}[U, 1, 1] == s, U], s] \\
&R = 2, v = 1, \theta = 1
\end{aligned}$$

(a)

$$\begin{array}{ll}
PostulateII & \frac{2^{2/3}}{3U^{2/3}} \\
PostulateIII & 2^{2/3}\left(\lambda^3\right)^{1/3} \\
PostulateIV & 0 \to \frac{3s^2}{4}
\end{array}$$

(b)

$$\begin{array}{ll}
PostulateII & \frac{2 * 2^{1/3}}{3U^{1/3}} \\
PostulateIII & 2^{1/3} * \lambda^{2/3} \\
PostulateIV & \left\{\left\{0 \to \frac{3\sqrt{s}}{2\sqrt{2}}\right\}\right\}
\end{array}$$

(c)

$$\begin{array}{ll}
PostulateII & \frac{1}{\sqrt{2(2+U)}} \\
PostulateIII & \sqrt{6}\sqrt{\lambda^2} \\
PostulateIV & \{\{0 \to s\}\}
\end{array}$$

(d)

$$\begin{array}{ll}
PostulateII & -\frac{4}{U^2} \\
PostulateIII & 4\lambda \\
PostulateIV & \left\{\left\{0 \to -\frac{4}{s^2}\right\}\right\}
\end{array}$$

(e)

$PostulateII$ $\frac{2*2^{3/5}U}{5\left(U^2\right)^{4/5}}$

$PostulateIII$ $2^{3/5}\left(\lambda^5\right)^{1/5}$

$PostulateIV$ $\left\{\left\{0\rightarrow-\frac{5s^{3/2}}{4\sqrt{2}}\right\},\left\{0\rightarrow\frac{5s^{3/2}}{4\sqrt{2}}\right\}\right\}$

(f)

$PostulateII$ $\frac{2}{U}$

$PostulateIII$ $-2\lambda Log[2]$

$PostulateIV$ $\left\{\left\{0\rightarrow e^{s/2}\right\}\right\}$

(g)

$PostulateII$ $\frac{1}{2\sqrt{eU}}$

$PostulateIII$ $\sqrt{\frac{2\lambda^2}{e}}$

$PostulateIV$ $\{\{0\rightarrow es\}\}$

(h)

$PostulateII$ $\frac{e^{-U/2}}{\sqrt{2U}}-\frac{e^{-U/2}\sqrt{U}}{\sqrt{2}}$

$PostulateIII$ $e^{-\lambda}\sqrt{2}$

$PostulateIV$ $\left\{\left\{0\rightarrow-\frac{2Log\left[-\frac{s^2}{2}\right]}{s\left(1+Log\left[-\frac{s^2}{2}\right]\right)}\right\}\right\}$

(i)

$PostulateII$ $\left\{\left\{0\rightarrow-\frac{2Log\left[\pm\frac{\sqrt{U}}{2\sqrt{2}}\right]}{U\left(1+Log\left[\pm\frac{\sqrt{U}}{2\sqrt{2}}\right]\right)}\right\}\right\}$

$PostulateIII$ $\frac{\sqrt{2}\lambda}{2}$

$PostulateIV$ $e^{-s/2}-e^{-s/2}(1+\frac{s}{2})$

(j)

$PostulateII$ $\left\{\left\{0\rightarrow-\frac{2Log\left[\pm\frac{U}{2e}\right]}{U\left(1+Log\left[\pm\frac{U}{2e}\right]\right)}\right\}\right\}$

$PostulateIII$ $\frac{3\lambda^2}{\sqrt{e}}$

$PostulateIV$ $e^{s/2}s+1/4e^{s/2}s^2$

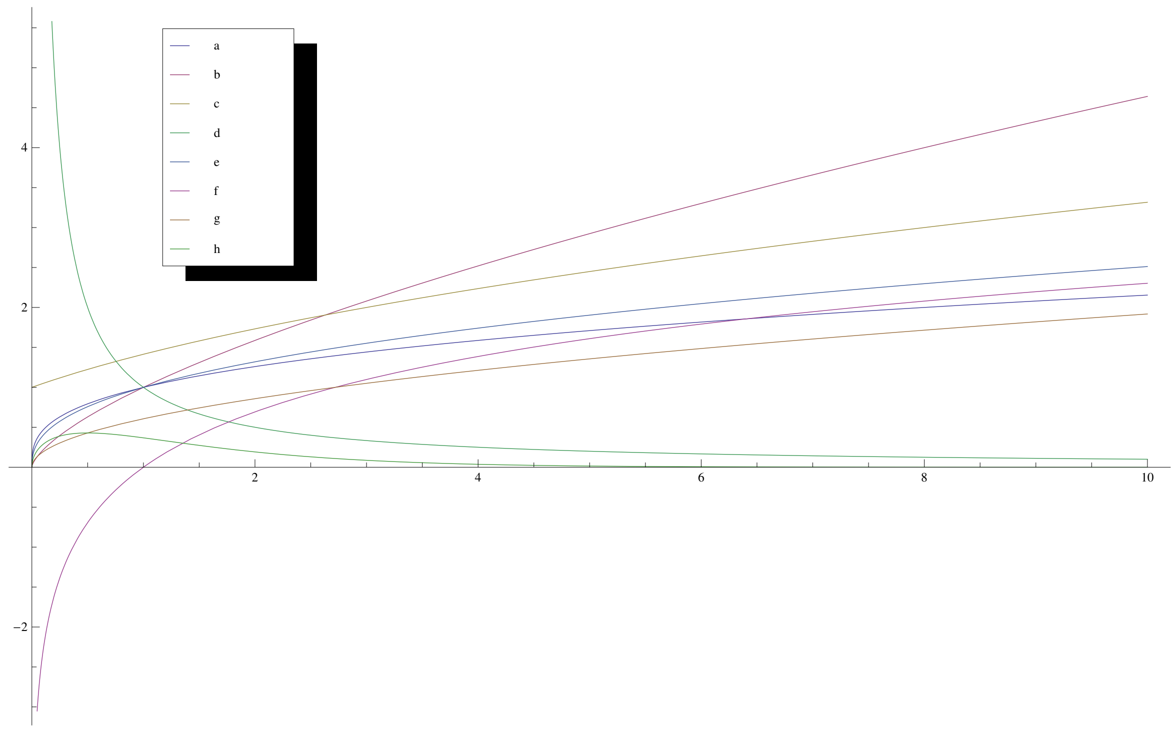

Figure 1: S vs. U, given relations A through H

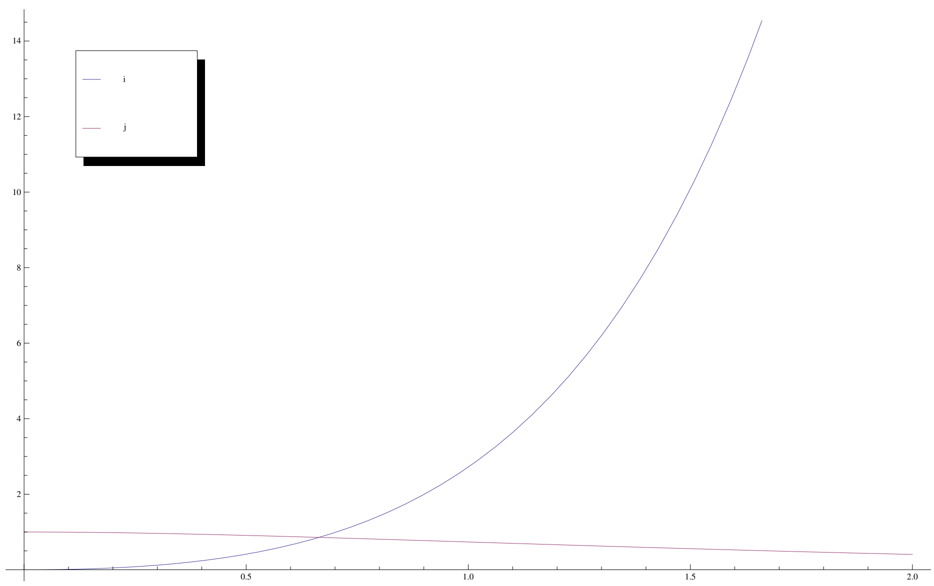

Figure 2: U vs. S, given relations I and J

Function	Postulate II	Postulate III	Postulate IV
A	✓	✓	✓
B	✓	X	✓
C	✓	✓	✓
D	X	✓	X
E	✓	✓	✓
F	✓	✓	X
G	✓	✓	✓
H	X	X	X
I	✓	✓	✓
J	X	X	✓

Plots

Results

$$\boxed{B, D, F, H, J \text{ are not physically permissible}}$$

Problem 5

Call

$$X = \frac{U_A}{U_A + U_B}$$

we know $U_A + U_B = 80$, therefore

$$U_A = 80X, \qquad U_B = 80\,(1 - X)$$

Then setting $R, v_0, \theta = 1$ and plugging in V_A, N_A

$$S_A = (N_A V_A U_A)^{1/3} = 0.129X^{1/3}$$

Similarly

$$S_B = (N_B V_B U_B)^{1/3} = 0.086(1 - X)^{1/3}$$

Summing to get S

$$S = 0.129X^{1/3} + 0.086(1 - X)^{1/3}$$

Entropy is maximized when $X = .65$, which is where we would expect the system to go to at equilibrium once the internal wall is made diathermal. When that happens

$$\boxed{X = 0.65,\ U_A = 80X = 52\ J}$$

$$\therefore \boxed{U_B = 28\ J}$$

An alternative non-graphical method is to solve for where

$$\frac{\partial S}{\partial U} = 0$$

Since

$$\frac{\partial S}{\partial U} = \frac{0.043}{x^{2/3}} - \frac{0.029}{(1 - x)^{2/3}} = 0$$

$$\boxed{X = 0.6475, \qquad U_A = 51.8, \qquad U_B = 28.2}$$

which is just a higher precision calculation than my first estimate from the plot.

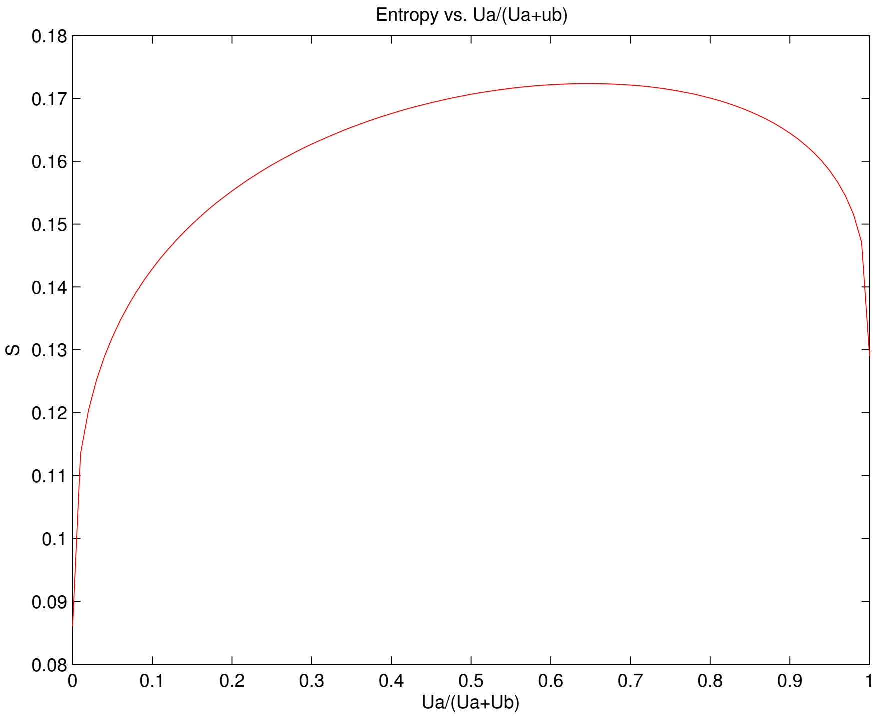

Figure 3: $S(X),\ X = \frac{U_A}{U_A+U_B}$

Problem 6

$$\frac{\partial}{\partial T}\frac{\partial}{\partial V}A = \frac{\partial}{\partial V}\frac{\partial}{\partial T}A$$
$$\frac{\partial}{\partial T}(-P) = \frac{\partial}{\partial V}(-S)$$

Problem 7

a)
$$\lambda^3$$

b)
$$\lambda^{1/2}$$

c)
$$\lambda^1$$

d)
$$\lambda^0$$

e) not homogenous