PHZ 3113, Mathematics Physics, Spring 2004

February 2^{th}, 2004

Assignment # 7:

due *Friday* February 6^{th} (at the beginning of class).

1. Derive by a method similar to that used in class that

$$-\int\int_{\text{A}} dx\,dy \frac{\partial P(x,y)}{\partial y} = \oint_{\text{C}} dx\,P(x,y),$$

where the path C is the rectangular path enclosing A and is traversed counter-clockwise, and A is the rectangular area used in class.

2. In electricity and magnetism, the energy density in an electromagnetic field is

$$\mathcal{E} = \frac{1}{2\epsilon_0}\mathbf{E}\cdot\mathbf{E} + \frac{1}{2\mu_0}\mathbf{B}\cdot\mathbf{B}.$$

If both $\mathbf{E}$ and $\mathbf{B}$ are functions of position in three dimensions *and* time, find the total differential $d\mathcal{E}$.

3. Boas, Ch.6, § 8, #17. Which, if either, of the two force fields

$$\mathbf{F}_1 = -y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}} + z\hat{\mathbf{k}}, \quad \mathbf{F}_2 = y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}} + z\hat{\mathbf{k}}$$

is conservative? Calculate for each field the work done in moving a particle counterclockwise around the circle $x = \cos(t)$, $y = \sin(t)$ in the (x, y) plane.

4. Find the work $\oint_C \mathbf{F}\cdot d\mathbf{r}$ done by a force $\mathbf{F} = y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}}$ in going all the way counterclockwise around a circle C given by $x^2 + y^2 + 2x = 0$, by the easiest technique you know.