Question 4:

For the machine component in Figure 4, determine:

1. The equation of motion of the system.
2. The natural frequency of the system if k is 120 N/m (the required material parameters are presented in Table 2).
3. Briefly discuss the effect of spring stiffness on the natural frequency for this system.
4. Evaluate the initial angular velocity of the system based on the measured evolution of angular displacement presented in Figure 5.
5. Determine the angular velocity and angular acceleration as a function of time for this system.
6. Hence, plot the resulting angular velocity and angular acceleration responses for the first three cycles using MS Excel.

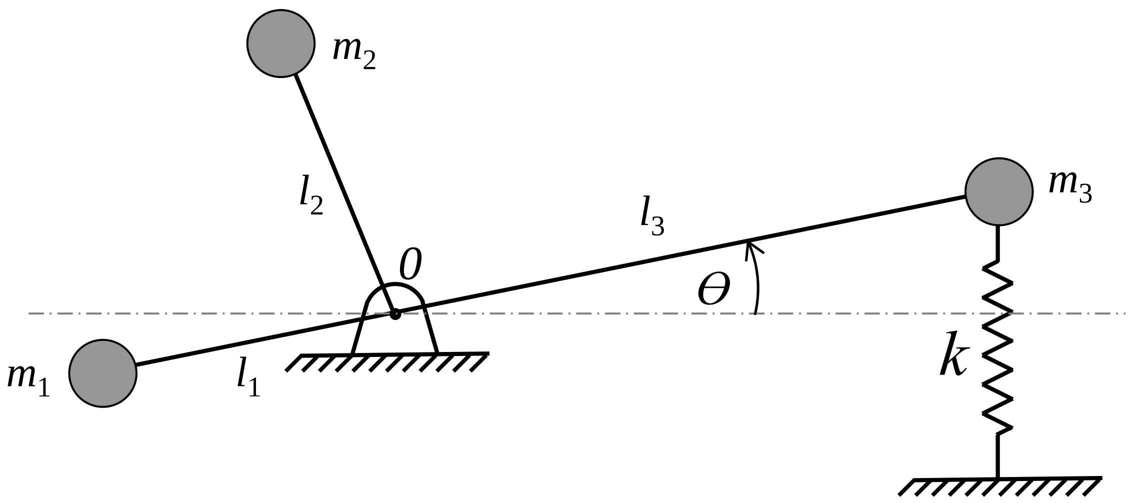

Figure 4: Machine component subjected to an angular displacement, θ.

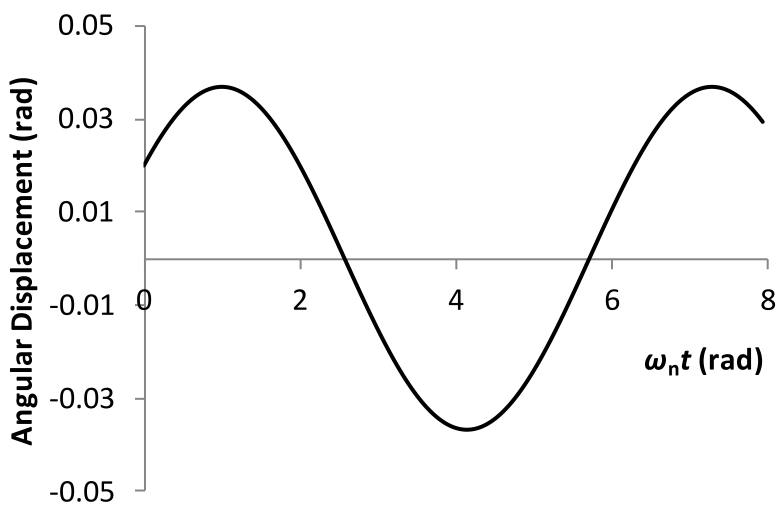

Figure 5: Measured response of the system in Question 4.

Table 2: Model constants for question 4.

m_1	20 kg	l_1	0.21 m
m_2	10 kg	l_2	0.27 m
m_3	25 kg	l_3	0.45 m

Question 5:

(a) A measurement device for monitoring the performance of a system has a mass, *m*, of 0.9 kg and is attached to a component operating at 2000 rpm, as illustrated in Figure 6. The measurement device is connected using a beam with an overall length of 350 mm, height of 10 mm and a width of 60 mm. If the beam material has a Young's modulus of 35 GPa, determine the natural frequency of the system.

(b) The beam material is replaced with a material which has a density of 7800 kg.m^{-3} and a Young's modulus of 165 GPa. Using Rayleigh's energy method, derive the following expression for the natural frequency of the new system:

$$\omega_n = \sqrt{\frac{3k}{3m + m_s}}$$

where k and m_s are the stiffness and mass of the stainless steel beam, respectively. Hence, determine the natural frequency of the new system.

Note: In your analysis, you may assume a linear relationship between the spring velocity and the mass velocity.

(c) Briefly discuss any potential problems with the modification to the system. Suggest possible solutions to address these problems.

(d) Comment on the importance of the mass of the beam member when evaluating the natural frequency of the two systems.

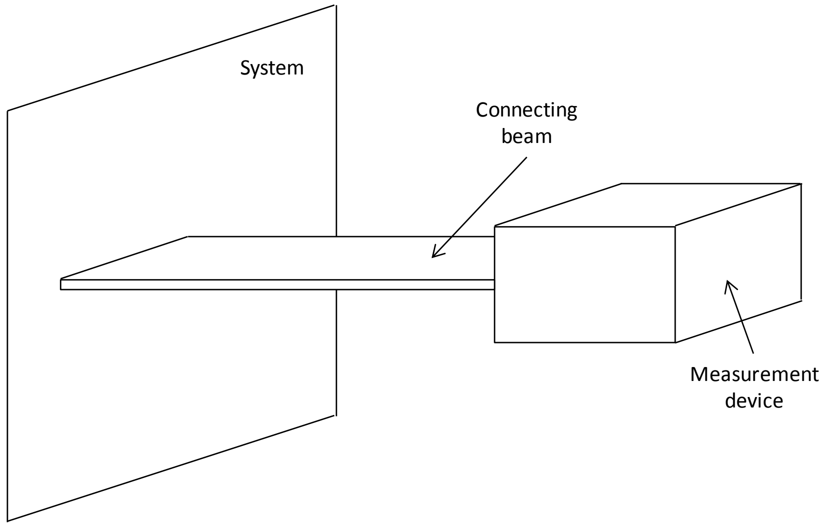

Figure 6: Schematic representation of the measurement device in Question 5.

Question 6

(a) A uniform rectangular cantilever beam of length l and mass m is fixed at one end as illustrated in Figure 7. If a mass M is applied at the free end, derive the following expression for the natural frequency of the system using Rayleigh's energy method:

$$\omega_n = \sqrt{\frac{3EI}{l^3 m_{\mathrm{eq}}}} = \sqrt{\frac{3EI}{l^3 (M + 0.23m)}}$$

where E is Young's modulus and I is the second moment of area.

(b) A mass of 1000 kg is attached to the end of a 1.6 m long cantilever beam, of depth 0.4 m and breadth 0.3 m. Determine the natural frequency of the system if the beam is manufactured from stainless steel with a Young's modulus of 206 GPa and a density of 7800 kg/m^3.

(c) In a bid to reduce the overall weight of the system, the manufacturer proposes replacing the steel beam by a carbon fibre composite beam with a Young's modulus of 70 GPa and density of 1600 kg/m^3. Determine a suitable beam depth for a square section cantilever beam of length 1.6 m, which will allow the system to be operated safely at a frequency of 40 Hz and factor of safety of 1.5.

Hint: In your analysis, assume that the beam has a mode shape given by the following function: $\lambda(c) = x_c(c)/x_l$ where $x_c(c)$ and x_l are the static deflections at any point c and at the end of the cantilever beam, respectively.

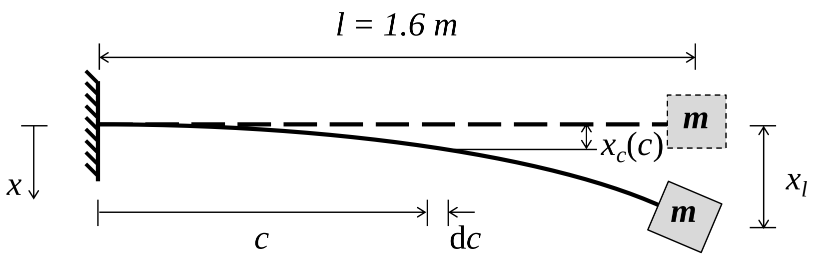

Figure 7: Geometry of the system for Question 6.