Let $x(t)$ be a function whose spectrum is limited to the range 0-W cps, whose sample values x_1, x_2, etc. (taken $1/2W$ sec apart) are nonzero only

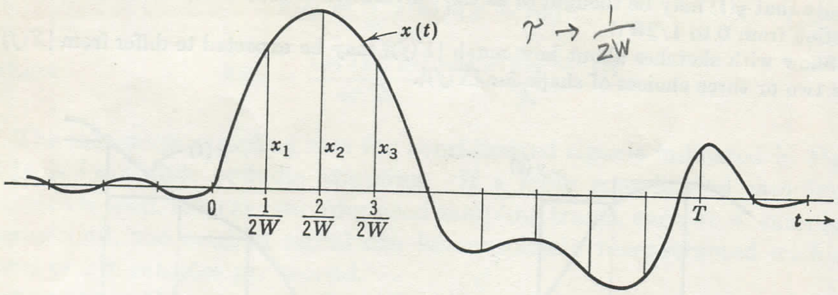

Fig. 2-15. Sampled band-limited signal with a finite number of nonzero samples.

in the range 0 to T (Fig. 2-15). It can be described exactly by[1]

$$x(t) = \sum_{n=1}^{2TW} x_n \frac{\sin \pi(2Wt - n)}{\pi(2Wt - n)} \tag{2-40}$$

This function can be represented as a vector in $2TW$-dimensional space whose components are

$$\mathbf{X} = x_1, x_2, x_3, \ldots, x_{2TW} \tag{2-41}$$

Addition and subtraction of multidimensional vectors may be defined by extension from the three-dimensional case; the dot product may be similarly defined:

$$\mathbf{X} + \mathbf{Y} \triangleq x_1 + y_1, x_2 + y_2, \ldots \tag{2-42}$$

$$\mathbf{X} \cdot \mathbf{Y} \triangleq (\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 + \cdots)\cdot(\mathrm{y}_1 + \mathrm{y}_2 + \mathrm{y}_3 + \cdots) = x_1y_1 + x_2y_2 + \cdots \tag{2-43}$$

The total energy in the signal $f(t)$ is, from Eq. (2-40),

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} \left[\sum_{n=1}^{2TW} x_n \frac{\sin \pi(2Wt - n)}{\pi(2Wt - n)}\right]^2 dt \tag{2-44}$$

[1] For simplicity in presentation, the signal-space concept is here discussed only in terms of band-limited signals restricted to the frequency range 0 to W cps. As indicated in Sec. 2-5, the sampling concept, and hence the signal-space representation, can be extended to band-limited signals in the range W_1 to $W_1 + W$. In some discussions later in the book it will be so used without further justification.

But note that[1]

$$\int_{-\infty}^{\infty} \frac{\sin \pi(2Wt - n)}{\pi(2Wt - n)} \frac{\sin \pi(2Wt - m)}{\pi(2Wt - m)}$$

Thus it follows that

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \frac{1}{2W} \sum_{n=1}^{2TW}$$

In a similar manner it follows that the do represents the integral of the product of

$$\boxed{\int_{-\infty}^{\infty} x(t)y(t)\,dt = \frac{1}{2W}\mathbf{X}\cdot\mathbf{Y}}$$

where x_n and y_n are the sample values Note that the time-average power of a si the sum of the squares of the sample valu of samples:

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt \approx \frac{1}{2TW}\mathbf{X}\cdot$$

$\vec{X}\cdot\vec{X} = X^2 = 2TWP$ so $X = \sqrt{2T}$

Thus all signals with the same average pow which terminate on a sphere (or hypersph

The sampling functions represent one terms of which $x(t)$ can be expanded. Fourier-series expansion.

Consider the set of band-limited tim whose sample values (taken $1/2W$ sec $(2\pi kt/T)$ in the interval $0 \le t \le T$ and ze sample values are sin $(2\pi kt/T)$ in this These are orthogonal, since (for $2TW > 2$

$$\int_{-\infty}^{\infty} c_j(t)c_k(t)\,dt = \frac{1}{2W}\sum_{n=1}^{2TW} \cos\frac{\pi jn}{TW}\ \mathrm{c}$$

[1] This is most easily shown using Parseval's the

$c_k = \cos(2\pi kt/T)$
$s_k = \sin(2\pi kt/T)$