"Mysteries" of Modern Physics and the Fundamental Constants c, h, and G

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Abstract. We review how the kinematic structures of special relativity and quantum mechanics both stem from the relativity principle, i.e., "no preferred reference frame" (NPRF). Essentially, NPRF applied to the measurement of the speed of light c gives the light postulate and leads to the geometry of Minkowski spacetime, while NPRF applied to the measurement of Planck's constant h gives "average-only" projection and leads to the qubit Hilbert space of quantum mechanics. These kinematic structures contain the counterintuitive aspects ("mysteries") of time dilation, length contraction, and quantum entanglement. In this essay, we extend the application of NPRF to the gravitational constant G and show that it leads to the "mystery" of the contextuality of mass in general relativity. Thus, we see an underlying coherence and integrity in modern physics via its "mysteries" and the fundamental constants c, h, and G.

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All undergraduate physics majors are shown how the counterintuitive aspects ("mysteries") of time dilation and length contraction in special relativity (SR) follow from the light postulate, i.e., that everyone measures the same value for the speed of light c, regardless of their motion relative to the source. And, we can understand the light postulate to follow from the relativity principle, sometimes referred to as "no preferred reference frame" (NPRF). Simply put, if the speed of light from a source was only equal to $c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$ (per Maxwell's equations) for one particular velocity relative to the source, that would certainly constitute a preferred reference frame [1, 2].

While time dilation and length contraction follow "analytically" from the light postulate, there are those who do not consider the light postulate explanatory, since it does not provide "hypothetically constructed" mechanisms to "synthetically" account for time dilation and length contraction [3,4]. That is, the postulates of SR are principles offered without corresponding "constructive efforts." In what follows, Einstein explains the difference between the two [5]:

> We can distinguish various kinds of theories in physics. Most of them are constructive. They attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out. [The kinetic theory of gases is an example.] ...
>
> Along with this most important class of theories there exists a second, which I will call "principle-theories." These employ the analytic, not the synthetic, method. The elements which form their basis and starting point are not hypothetically constructed but empirically discovered ones, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy. [Thermodynamics is an example.] ...
>
> The advantages of the constructive theory are completeness, adaptability, and clearness, those of the principle theory are logical perfection and security of the foundations. The theory of relativity belongs to the latter class.

Here is why Einstein formulated SR as a principle theory [6, pp. 51-52]:

> By and by I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts. The longer and the more despairingly I tried, the more I came to the conviction that only the discovery of a universal formal principle could lead us to assured results.

Despite the fact that "there is no mention in relativity of exactly *how* clocks slow, or *why* meter sticks shrink" (no "constructive efforts"), the "empirically discovered" principles of SR are so compelling that "physicists always seem so sure about the particular theory of Special Relativity, when so many others have been superseded in the meantime" [7].

As it turns out, we are in a similar position today with quantum mechanics (QM). For example, QM accurately predicts violations of the Clauser-Horne-Shimony-Holt

(CHSH) inequality all the way to the Tsirelson bound for Bell state entanglement without providing a corresponding constructive account. This leads some to believe that QM and SR are fundamentally incompatible [8]. Bell himself voiced concerns about the compatibility of SR and QM based on quantum entanglement [9, p. 172]:

> For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory.

Quantum entanglement leads others to believe QM is "incomplete" or even "wrong." For example, Smolin writes [10, p. xvii]:

> I hope to convince you that the conceptual problems and raging disagreements that have bedeviled quantum mechanics since its inception are unsolved and unsolvable, for the simple reason that the theory is wrong. It is highly successful, but incomplete.

Of course, this is precisely the complaint leveled by Einstein, Podolsky, and Rosen (EPR) in their famous paper, "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" [11]. The EPR paper was published in 1935 and yet physics still has no constructive account of quantum entanglement. Thus, like Einstein with SR, physicists are starting to despair of finding a causal mechanism responsible for quantum entanglement. Hardy writes [12, p. 224]:

> The standard axioms of [quantum theory] are rather ad hoc. Where does this structure come from? Can we write down natural axioms, principles, laws, or postulates from which we can derive this structure? Compare with the Lorentz transformations and Einstein's two postulates for special relativity. Or compare with Kepler's Laws and Newton's Laws. The standard axioms of quantum theory look rather ad hoc like the Lorentz transformations or Kepler's laws. Can we find a natural set of postulates for quantum theory that are akin to Einstein's or Newton's laws?

Along these lines, we recently showed that in fact the qubit Hilbert space structure at the foundation of Hardy's and Dakic & Brukner's reconstructions of quantum theory [12,13] follows from the relativity principle (NPRF) [14–16]. We did this by extending NPRF to include the measurement of another fundamental constant of nature, Planck's constant h.

As Weinberg points out, measuring an electron's spin via Stern-Gerlach (SG) magnets constitutes the measurement of "a universal constant of nature, Planck's constant" [17] (Figure 1). So if NPRF applies equally here, everyone must measure the same value for Planck's constant h regardless of their SG magnet orientations relative to the source, which like the light postulate is an "empirically discovered" fact. By "relative to the source," we might mean relative "to the vertical in the plane perpendicular to the line of flight of the particles [18, p. 943]" ($\hat{z}$ in Figure 1, for example). Thus, different

SG magnet orientations relative to the source constitute different "reference frames" in QM just as different velocities relative to the source constitute different "reference frames" in SR. Spatial rotations and Lorentz boosts form the Lorentz transformations of course.

If we create a preparation state oriented along the positive z axis as in Figure 2, i.e., $|\psi\rangle = |u\rangle$, our spin angular momentum is $\vec{S} = +1\hat{z}$ (in units of $\frac{\hbar}{2} = 1$). Now proceed to make a measurement with the SG magnets oriented at $\hat{b}$ making an angle β with respect to $\hat{z}$ (Figure 2). According to classical physics (Figure 3), we expect to measure $\vec{S} \cdot \hat{b} = \cos(\beta)$ (Figure 4), but we cannot measure anything other than ± 1 due to the relativity principle (contra the prediction by classical physics). As a consequence, we can only recover $\cos(\beta)$ *on average* (Figure 5), i.e., NPRF dictates "average-only" projection

$$(+1)P(+1 \mid \beta) + (-1)P(-1 \mid \beta) = \cos(\beta) \tag{1}$$

Solving simultaneously with our normalization condition $P(+1 \mid \beta) + P(-1 \mid \beta) = 1$, we obtain the quantum state

$$P(+1 \mid \beta) = \cos^2\left(\frac{\beta}{2}\right) \tag{2}$$

and

$$P(-1 \mid \beta) = \sin^2\left(\frac{\beta}{2}\right) \tag{3}$$

This explains the ineluctably probabilistic nature of QM, as pointed out by Mermin [19, p. 10]:

> Quantum mechanics is, after all, the first physical theory in which probability is explicitly not a way of dealing with ignorance of the precise values of existing quantities.

This means that QM is as "complete" as possible, given NPRF. Of course, these "average-only" results due to "no fractional outcomes per NPRF" hold precisely for the qubit Hilbert space structure of QM whence Hardy's and Dakic & Brukner's reconstructions of quantum theory.

This analysis extends to "average-only" conservation for a pair of spin entangled particles. Specifically, when Alice and Bob make their SG spin measurements at the same angle in the plane of symmetry (same reference frame, Figure 6), conservation of spin angular momentum dictates that they obtain the same result (both $+1$ or both -1) for the spin triplet states (opposite results for the spin singlet state). Thus again, classical physics suggests that if Bob makes his SG spin measurement at angle θ with respect to Alice (different reference frames), then according to Alice he should obtain $\cos(\theta)$ when she obtains $+1$ in accord with the conservation of "inherent/intrinsic" angular momentum (Figure 7). But, Bob can only ever measure ± 1 per the relativity principle, just like Alice, so the conservation principle is constrained to hold only *on average* per NPRF (Figures 8 & 9). Thus, Bell state entanglement and the Tsirelson bound are "mysteries" precisely because of "average-only" conservation, which follows from "average-only" projection, which follows from the relativity principle, so this is conservation per NPRF (Figure 10). Consequently, we see that the relativity principle reveals an underlying coherence between (non-relativistic) QM and SR (Figure 11) where others have perceived tension [8, 9].

Given this result, one immediately wonders if general relativity (GR) can be brought into the mix via the relativity principle and the gravitational constant G. Of course it can and the associated counterintuitive aspect ("mystery") in GR is the contextuality of mass. We have already shown in previous Gravity Research Foundation essays [20, 21] how this might resolve the missing mass problem without having to invoke non-baryonic dark matter.

Specifically, we are pointing out the well-known result per GR that matter can simultaneously possess different values of mass when it is responsible for different combined spatiotemporal geometries. Here "reference frame" refers to each of the different spatiotemporal geometries associated with one and the same matter source. Thus, we have extended the relativity principle from the Lorentz transformations of flat spacetime to the general coordinate transformations of curved spacetime. Tacitly assumed in this result is of course that G has the same value in each reference frame, which is consistent with NPRF as applied to c and h above. This spatiotemporal contextuality of mass is not present in Newtonian gravity where mass is an intrinsic property of matter.

For example, when a Schwarzschild vacuum surrounds a spherical matter distribution the "proper mass" M_p of the matter, as measured locally in the matter, can be different than the "dynamic mass" M in the Schwarzschild metric responsible for orbital kinematics about the matter [22, p. 126]. This difference is attributed to binding energy and goes as $dM_p = \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1/2} dM$. In another example, suppose a Schwarzschild vacuum surrounds a sphere of Friedmann-Lemaître-Robertson-Walker (FLRW) dust, as used originally to model stellar collapse [23, pp. 851-853]. The dynamic mass M of the surrounding Schwarzschild metric is related to the proper mass M_p of the FLRW dust, as joined at FLRW radial coordinate χ_o, by

$$\frac{M_p}{M} = \frac{3(2\chi_o - \sin(2\chi_o))}{4\sin^3(\chi_o)} \tag{4}$$

where

$$ds^2 = -c^2 d\tau^2 + a^2(\tau)\left(d\chi^2 + \sin^2\chi d\Omega^2\right) \tag{5}$$

is the closed FLRW metric [24]. We should quickly point out that this may prima facie seem to constitute a violation of the equivalence principle, as understood to mean inertial mass equals gravitational mass, since inertial mass can't be equal to two different values of gravitational mass. But, the equivalence principle says simply that spacetime is locally flat [25, pp. 68-69] and that is certainly not being violated here nor with any solution to Einstein's equations.

Thus, contrary to what many believe about SR, QM, and GR collectively, these theories are comprehensive (not "incomplete" per [10] and [11]) and coherent (not "in conflict" per [8] and [9]). In order to appreciate the beauty of these theories collectively, one need only view them per the relativity principle (NPRF) with their associated "mysteries" corresponding to the fundamental constants c, h, and G, respectively.

FIGURES

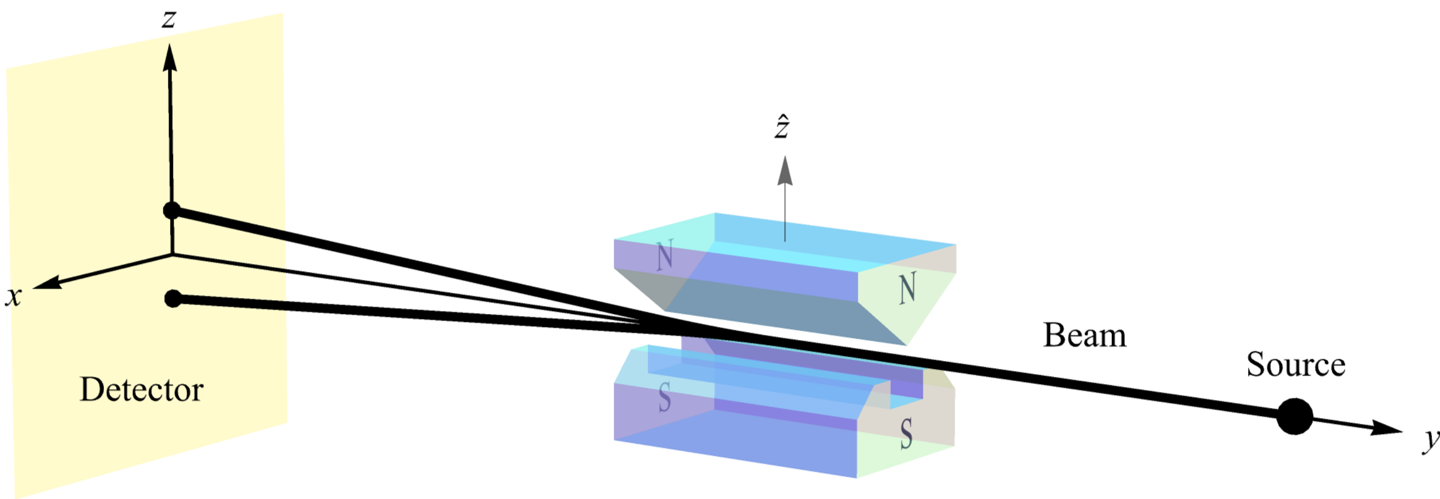

Figure 1: A Stern-Gerlach (SG) spin measurement showing the two possible outcomes, up $(+\frac{\hbar}{2})$ and down $(-\frac{\hbar}{2})$ or +1 and −1, for short.

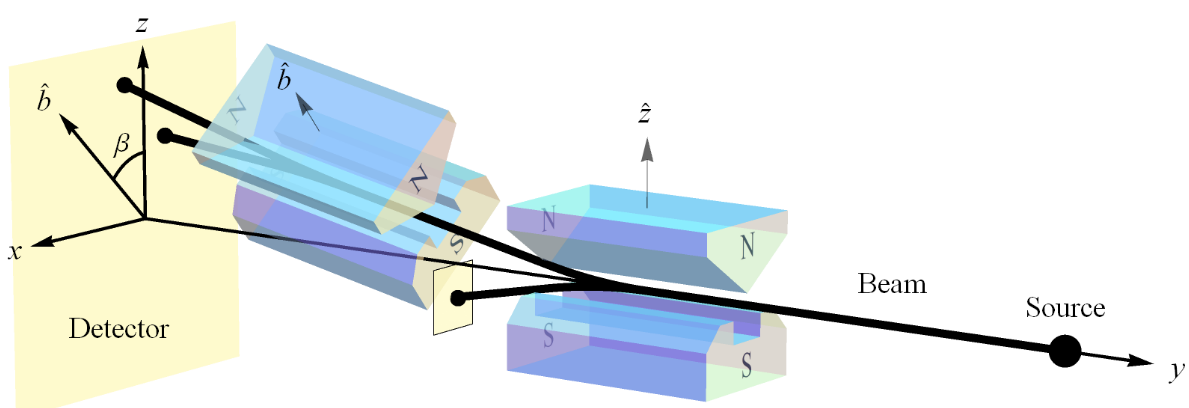

Figure 2: In this set up, the first SG magnets (oriented at $\hat{z}$) are being used to produce an initial state $|\psi\rangle = |u\rangle$ for measurement by the second SG magnets (oriented at $\hat{b}$).

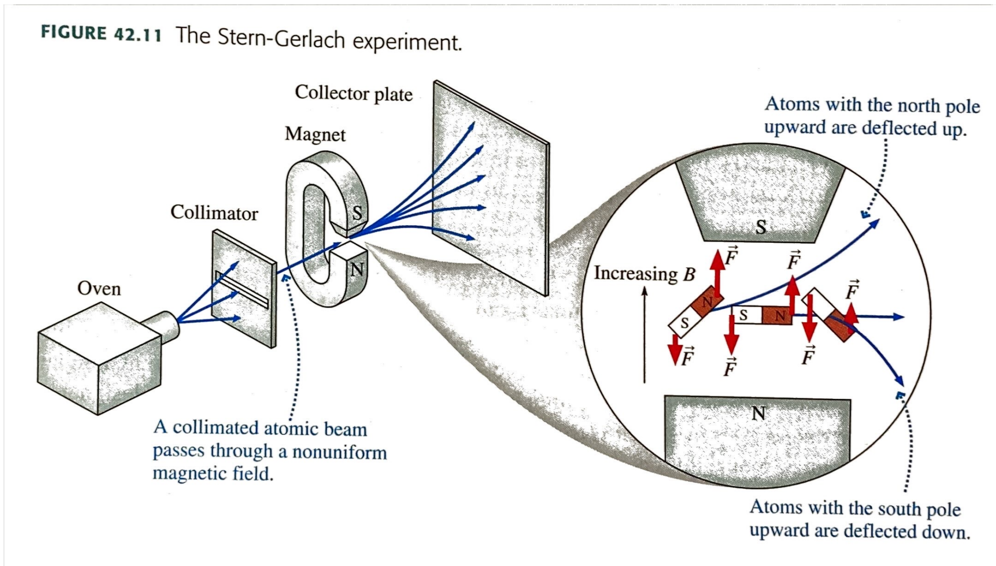

Figure 3: **The classical constructive model of the Stern-Gerlach experiment.** If the atoms enter with random orientations of their "inherent/intrinsic" magnetic moments (due to their "inherent/intrinsic" angular momenta), the SG magnets should produce all possible deflections, not just the two that are observed. The difference between the classical prediction and the quantum reality uniquely distinguishes the quantum joint distribution from the classical joint distribution for the Bell spin states [26]. Figure reproduced from Knight [2, p. 1307].

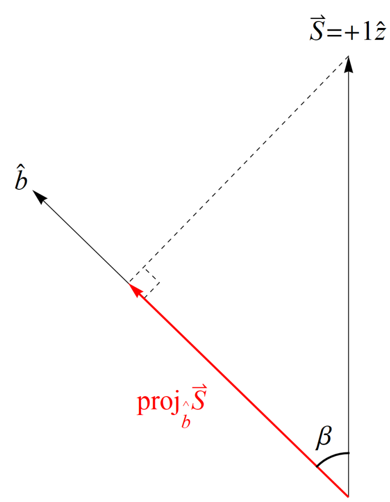

Figure 4: The "intrinsic" angular momentum of Bob's particle $\vec{S}$ projected along his measurement direction $\hat{b}$. This does *not* happen with spin angular momentum due to the relativity principle (NPRF).

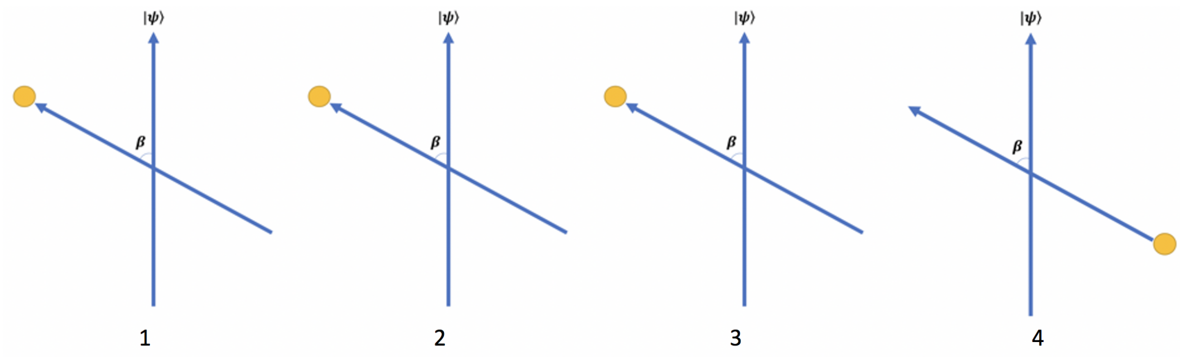

Figure 5: An ensemble of 4 SG measurement trials for $\beta = 60°$ in Figure 2. The tilted blue arrow depicts an SG measurement orientation and the vertical arrow represents our preparation state $|\psi\rangle = |u\rangle$. The yellow dots represent the two possible measurement outcomes for each trial, up (located at arrow tip) or down (located at bottom of arrow). The expected projection result of $\cos(\beta)$ cannot be realized because the measurement outcomes are binary (quantum) with values of $+1$ (up) or -1 (down) per NPRF. Thus, we have "average-only" projection for all 4 trials (three up outcomes and one down outcome average to $\cos(60°) = \frac{1}{2}$). That is, the *average* of the ± 1 outcomes equals the projection of the initial spin angular momentum vector $\vec{S} = +1\hat{z}$ in the measurement direction $\hat{b}$, i.e., $\vec{S} \cdot \hat{b} = \cos(60°) = \frac{1}{2}$.

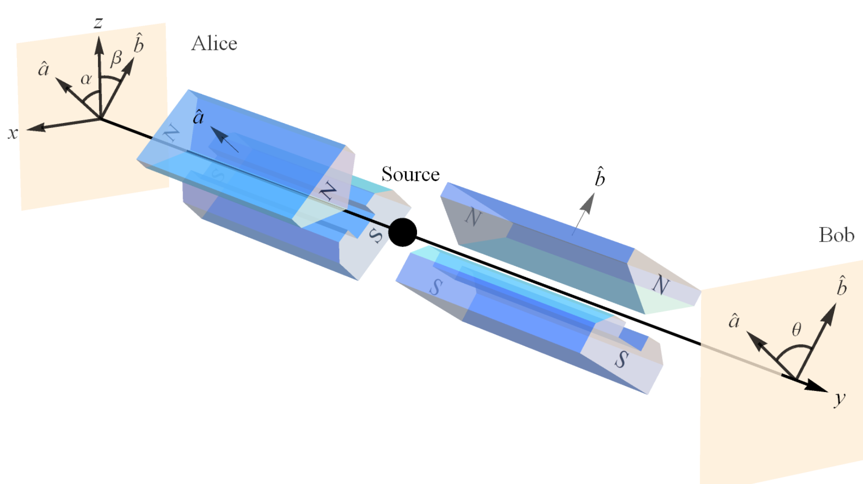

Figure 6: Alice and Bob making spin measurements on a pair of spin-entangled particles with their Stern-Gerlach (SG) magnets and detectors in the xz-plane. Here Alice and Bob's SG magnets are not aligned so these measurements represent different reference frames.

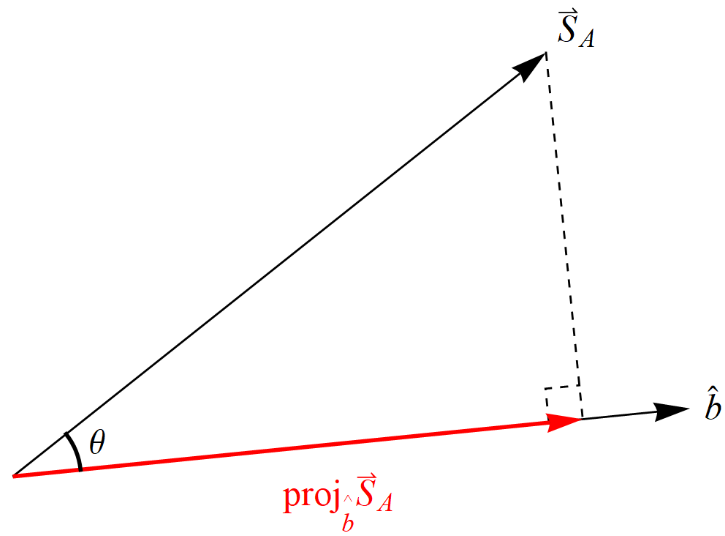

Figure 7: The "intrinsic" angular momentum of Bob's particle $\vec{S}_B = \vec{S}_A$ projected along his measurement direction $\hat{b}$. This does *not* happen with spin angular momentum because Bob must always measure ± 1, no fractions, in accord with the relativity principle (NPRF).

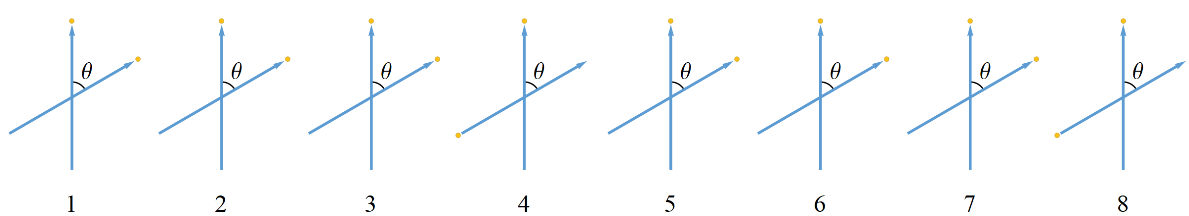

Figure 8: An ensemble of 8 SG measurement trials of a spin triplet state showing Bob's(Alice's) outcomes corresponding to Alice's(Bob's) +1 outcomes when $\theta = 60°$ (Figure 6). Spin angular momentum is not conserved in any given trial, because there are two different measurements being made, i.e., outcomes are in two different reference frames, but it is conserved *on average* for all 8 trials (six up outcomes and two down outcomes average to $\cos(60°) = \frac{1}{2}$). It is impossible for spin angular momentum to be conserved explicitly in each trial since the measurement outcomes are binary (quantum) with values of +1 (up) or −1 (down) per NPRF.

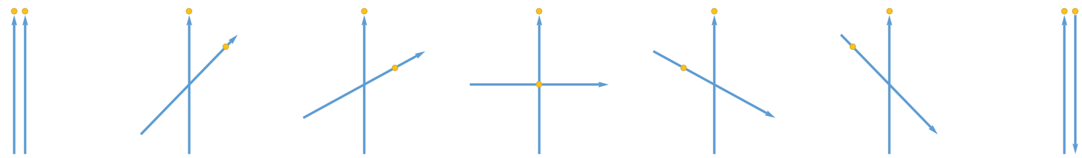

Figure 9: **Average View for the Spin Triplet States**. Reading from left to right, as Bob rotates his SG magnets relative to Alice's SG magnets for her +1 outcome, the average value of his outcome varies from +1 (totally up, arrow tip) to 0 to −1 (totally down, arrow bottom). This obtains per conservation of spin angular momentum on average in accord with the relativity principle (NPRF). Bob can say exactly the same about Alice's outcomes as she rotates her SG magnets relative to his SG magnets for his +1 outcome. That is, their outcomes can only satisfy conservation of spin angular momentum on average in different reference frames, because they only measure ±1, never a fractional result. Again, just as with the light postulate of SR, we see that the relativity principle leads to a counterintuitive result. Here it requires quantum outcomes $\pm 1\left(\frac{\hbar}{2}\right)$ for all measurements leading to the "mystery" of "average-only" conservation.

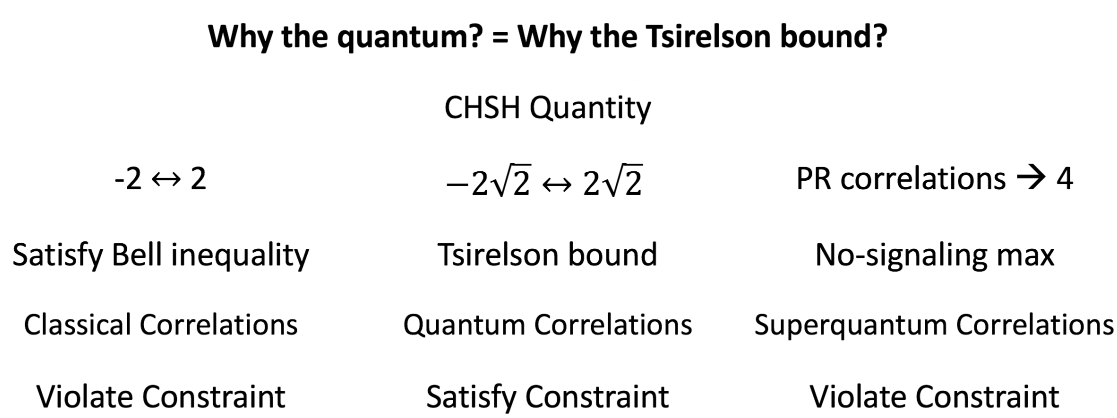

Figure 10: Bub's version of Wheeler's question "Why the quantum?" is "Why the Tsirelson bound?" The "constraint" is conservation per NPRF.

Special Relativity	Quantum Mechanics
Empirical Fact: Alice and Bob both measure *c*, regardless of their motion relative to the source.	Empirical Fact: Alice and Bob both measure $\pm 1 \left(\frac{\hbar}{2}\right)$, regardless of their SG orientation relative to the source.
Alice(Bob) says of Bob(Alice): Must correct time and length measurements.	Alice(Bob) says of Bob(Alice): Must average results.
NPRF: Relativity of simultaneity.	NPRF: Relativity of data partition.

Figure 11: **Comparing SR with QM according to the relativity principle, "no preferred reference frame" (NPRF)**. Because Alice and Bob both measure the same speed of light c, regardless of their motion relative to the source per NPRF, Alice(Bob) may claim that Bob's(Alice's) length and time measurements are erroneous and need to be corrected (length contraction and time dilation). Likewise, because Alice and Bob both measure the same values for spin angular momentum $\pm 1 \left(\frac{\hbar}{2}\right)$, regardless of their SG magnet orientation relative to the source per NPRF, Alice(Bob) may claim that Bob's(Alice's) individual ± 1 values are erroneous and need to be corrected (averaged, Figures 8 & 9). In both cases, NPRF resolves the "mystery" it creates. In SR, the apparently inconsistent results can be reconciled via the relativity of simultaneity. That is, Alice and Bob each partition spacetime per their own equivalence relations (per their own reference frames), so that equivalence classes are their own surfaces of simultaneity and these partitions are equally valid per NPRF. This is completely analogous to QM, where the apparently inconsistent results per the Bell spin states arising because of NPRF can be reconciled by NPRF via the "relativity of data partition." That is, Alice and Bob each partition the data per their own equivalence relations (per their own reference frames), so that equivalence classes are their own $+1$ and -1 data events and these partitions are equally valid.

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