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Gravity as Quantum Entanglement Force

Jae-Weon Lee*

*Department of energy resources development, Jungwon University,
5 dongburi, Goesan-eup, Goesan-gun Chungbuk Korea 367-805*

Hyeong-Chan Kim[†]

Division of Liberal Arts, Chungju National University, Chungju 380-702, Republic of Korea

Jungjai Lee[‡]

Department of Physics, Daejin University, Pocheon, Gyeonggi 487-711, Korea

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We conjecture that quantum entanglement of matter and vacuum in the universe tend to increase with time, like entropy, and there is an effective force called quantum entanglement force associated with this tendency. It is also suggested that gravity and dark energy are types of the quantum entanglement force, similar to Verlinde's entropic force. If the entanglement entropy of the universe saturates the Bekenstein bound, this gives holographic dark energy with the equation of state consistent with current observational data. This connection between quantum information and gravity gives some new insights on the origin of gravity, dark energy, the holographic principle and arrow of time.

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*scikid@gmail.com

[†]hyeongchan@gmail.com

[‡]jjlee@daejin.ac.kr

I. INTRODUCTION

Recently proposed Verlinde's idea [1] linking gravitational force to entropic force attracted much attention [2–9]. He derived the Newton's equation and the Einstein equation from this relation. Padmanabhan also proposed a similar idea [10]. However, Verlinde's proposal is based on some rather unusual assumptions such as the proportionality of the holographic entropy on the distance, and the holographic principle holding on equipotential surfaces.

In this paper, we conjecture that in general quantum entanglement of matter or the vacuum in the universe increases like the entropy and there is a new kind of force ('quantum entanglement force', henceforth) associated with this tendency similar to the entropic force. (This force is different from the 'entanglement force' of polymer science.) From this perspective it is also suggested that gravity and dark energy are types of the quantum entanglement force associated with the increase of the entanglement, similar to Verlinde's entropic force linked to the second law of thermodynamics. Our model relies on the well established quantum entanglement theory and uses less assumptions.

In a series of works [11–15] we have investigated the quantum informational nature of gravity, using especially the quantum entanglement and the Landauer's principle. Using the concepts, we suggested that dark energy is related to the quantum entanglement of the vacuum fluctuation [11] at a cosmic horizon [16, 17], and derived the first law of black hole thermodynamics from the second law of thermodynamics [12]. Recently, we also suggested [14, 15] that classical Einstein gravity can be derived by considering quantum entanglement entropy and an information erasure at Rindler horizons and Jacobson's idea linking the Einstein equation to thermodynamics. All our results imply that gravity has something to do with quantum information, especially quantum entanglement.

In Sec. II we discuss the relation between entanglement and the holographic principle. In Sec. III we introduce the concept of quantum entanglement force and suggest that gravity is a quantum entanglement force. In Sec. VI the predictions of our dark energy theory are compared with the recent observational data. Section V contains discussions.

II. ENTANGLEMENT AND HOLOGRAPHY

In quantum information science, quantum entanglement is a central concept and a precious resource allowing various quantum information processing such as quantum key distribution. The entanglement is a quantum nonlocal correlation which can not be prepared by local operations and classical communication. For pure states the entanglement entropy S_{Ent} is a good measure of entanglement. For a bipartite system AB described by a full density matrix ρ_{AB} , S_{Ent} is the von Neumann entropy $S_{Ent} = -Tr(\rho_A \ln \rho_A)$ for a reduced density matrix $\rho_A \equiv Tr_B \rho_{AB}$ obtained by partial tracing part B.

As a typical example in quantum field theory, consider a massless scalar field ϕ in a flat spacetime with Hamiltonian [18]

$$H = \int d^3x (|\nabla\phi(x)|^2 + |\pi(x)|^2), \quad (1)$$

where $\pi(x)$ is the momentum of the field. For a spherical region as shown in Fig. 1, one can expand the field with spherical harmonics on a discrete radial coordinate with a UV-cutoff to obtain an effective Hamiltonian for discretized field oscillators ϕ_{lmj} [18], which contains terms like $\phi_{lmj} * \phi_{lm(j+1)}$, where lmj are angular and radial indices. These terms represent a nearest neighbor interaction along the r direction. One can find similar terms generating entanglement between two regions for more general spacetime and fields. The entanglement of the generic quantum field vacuum was also shown by using the Reeh-Schlieder theorem [19, 20].

In general, the vacuum entanglement entropy of a spherical region with a radius r with quantum fields can be expressed in the form

$$S_{ent} = \frac{\beta r^2}{b^2}, \quad (2)$$

where β is an $O(1)$ constant that depends on the nature of the field and b is the UV-cutoff. By performing numerical calculations on a sphere lattice Srednicki obtained a value $\beta = 0.3$ for the massless field. A similar value was obtained for the entanglement entropy for a massless scalar field in the Friedmann universe in Ref. [21, 22]. More generally we may add up contributions from other fields with $\beta = \beta_j$ [21]. If there are N_j spin degrees of freedom of the j -th field, this implies

$$S_{ent} = \sum_j \beta_j N_j \frac{r^2}{b^2} \equiv \frac{\alpha r^2}{l_P^2}. \quad (3)$$

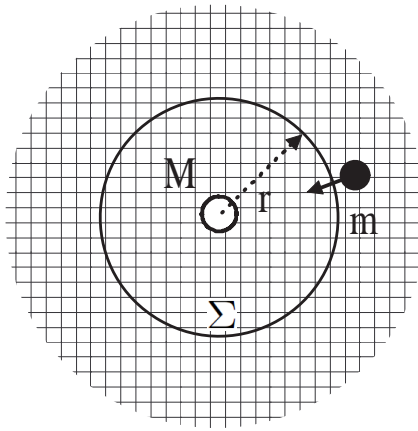


FIG. 1. The space around a massive object with mass M can be divided into two subspaces, the inside and the outside of an imaginary spherical surface with a radius r . The surface Σ has the entanglement entropy $S_{ent} \propto r^2$ and entanglement energy $E_{ent} \equiv \int_{\Sigma} T_{ent} dS_{ent}$. If there is a test particle with mass m , it feels an effective attractive force in the direction of increase of entanglement.

where the Planck length $l_P = \sqrt{\hbar G/c^3}$. If we choose $b = 1/M_P$, where M_P is the reduced Planck mass, then

$$\alpha = \frac{1}{8\pi} \sum_j \beta_j N_j. \quad (4)$$

It will be very important to obtain the value of α by explicit calculation in the future. It was conjectured that the Bekenstein-Hawking Entropy

$$S_{BH} = \frac{c^3 A}{4G\hbar} \quad (5)$$

saturates the information bound that a region of space with a surface area A can contain [23]. If S_{ent} saturates this bound, i.e., $S_{ent} = S_{BH}$, then $\alpha = \pi$.

Why are we considering the quantum entanglement as an essential concept for gravity? First, there are interesting similarities between the holographic entropy and the entanglement entropy of a given surface. Both are proportional to its area in general and related to quantum nonlocality. Second, when there is a gravitational force, there is always a Rindler horizon for some observers, which acts as information barrier for the observers. This can lead to ignorance of information beyond the horizons, and the lost information can be described by the entanglement entropy [14]. The spacetime should bend itself so that the increase of the entanglement entropy compensate the lost information of matter. In [14] we suggested that the Einstein equation is just an equation describing this relation. Third, if we use the entanglement entropy of quantum fields instead of thermal entropy of the holographic screen, we can understand the microstates of the screen and explicitly calculate, in principle, relevant physical quantities using the quantum field theory in the curved spacetime. The microstates can be thought of as just quantum fields on the surface or its discretized oscillators. Finally, identifying the holographic entropy as the entanglement entropy could explain why the derivations of the Einstein equation is involved with entropy, the Planck constant \hbar and, hence, quantum mechanics. All these facts indicate that quantum mechanics and gravity has an intrinsic connection, and the holographic principle itself has something to do with quantum entanglement.

III. ENTANGLEMENT ENERGY AND ENTANGLEMENT FORCE

Separable (i.e., not entangled) states are fragile in the sense that the states can be easily entangled with environments surrounding the states. A well known example is the Schrödinger cat paradox. In the paradox, no matter how well we separate a box which contains the cat from its environment, one can not fully block the information leakage of the cat toward the environment outside the box. Thus, even if we carefully prepared the superposition of the cat's state $|dead\ cat\rangle + |live\ cat\rangle$, the state easily gets entangled with the environment to be $|dead\ cat\rangle|env_0\rangle + |live\ cat\rangle|env_1\rangle$, where $|env_0\rangle$ and $|env_1\rangle$ represent the corresponding states of the environment. This entanglement between the cat and the environment induces 'decoherence' of the cat's effective density matrix when we trace out the environment states.

Although this process is *reversible* in principle, practically the reverse process is hardly observable in the macroscopic scale. Ironically, this is one of the reasons why it is so hard to observe controllable quantum entanglement in a laboratory and to build a practical quantum computer using the states. The quantum system of interest gets so easily entangled with its environment uncontrollably and loses coherence within the system. Decoherence is also related to the emergence of the classical world from the quantum world [24].

It might seem strange that the entanglement in the universe is increasing in general, considering that quantum evolution of a density matrix is described by using a unitary matrix U as $\rho \rightarrow U^\dagger \rho U$, which is reversible. This paradoxical situation is very similar to the case with the entropy; Even though the Schrödinger equation and the Einstein equation is time reversible, we see many time irreversible phenomena in the macroscopic world, and total entropy always does not decrease. One way of handling this ‘Loschmidt’s paradox’ is assuming that the early universe had a very small entropy due to, for example, inflation. Similarly we can assume that the early universe began with very small entanglement too. Thus, we can expect that the universe has a strong tendency to increase entanglement among its constituents (matter, quantum field, spacetime) as well as entropy. This might give us some new insights on the issues of arrow of time. The direction of the time (i.e., arrow of time) seems to be the direction in which the total entanglement of the universe increases. That is,

$$\frac{dS_{ent}}{dt} \geq 0 \quad (6)$$

for a sufficiently large macroscopic system and its environment. In [13] we argued that the time evolution of the universe is related to the expansion of the cosmic event horizon and its entanglement entropy.

Then, what is the relation between gravity and the entanglement? In [11], it was pointed out that a cosmic horizon has a kind of thermal energy called entanglement energy related to S_{ent}

$$dE_{ent} \equiv k_B T_{ent} dS_{ent}, \quad (7)$$

and suggested that it is the origin of dark energy. (Here, we explicitly denoted the Boltzman constant k_B for clearness.) This energy can be interpreted as effective energy obtained by tracing out the Hilbert space describing the outside of the horizon. It is also the energy of the vacuum fluctuation around the horizon. In [14] we pointed out that this energy is very similar to the equipartition energy of the horizon [1, 25]. If this energy is a function of parameters r_i , one can define a generalized force

$$F_{ent,i} \equiv \frac{dE_{ent}}{dr_i} = k_B T_{ent} \partial_{r_i} S_{ent}, \quad (8)$$

which is similar to entropic force. We call this force a ‘quantum entanglement force’.

Now, we conjecture that quantum entanglement of matter and vacuum in the universe tend to increase over time like entropy, and that gravity is a kind of this quantum entanglement force, similar to Verlinde’s entropic force.

From now on, we reinterpret the Verlinde’s theory in terms of our entanglement theory. To do this we consider a situation in Fig. 1. First, one can integrate Eq. (7) on the *isothermal* spherical surface Σ with radius r surrounding a mass M

$$E_{ent} = \int_{\Sigma} dE_{ent} = k_B T_{ent} \int_{\Sigma} dS_{ent} = \frac{\hbar G M}{2\pi c r^2} \frac{\alpha r^2}{l_P^2} = \frac{\alpha M c^2}{2\pi}, \quad (9)$$

where we have used the Unruh temperature

$$T_U = \frac{\hbar a}{2\pi c k_B} = \frac{\hbar G M}{2\pi c k_B r^2} \quad (10)$$

for T_{ent} . In [14] we identified T_U as the Rindler horizon temperature observed by a test particle under the influence of the mass M . For the holographic condition $E = M c^2$ to hold on the surface, α should be 2π , which exceeds the Bekenstein bound. This discrepancy could be removed by using the relation $E = 2k_B T S$ by Padmanabhan [26], which seems to be valid when there is an active gravitational mass. In that case we recover $\alpha = \pi$.

Following the Verlinde’s approach, we express the tendency of maximizing entanglement by the condition

$$\frac{dS_{ent}(E_{ent} + e^{V(r)} m, r)}{dr} = \partial_r S_{ent} + \frac{\partial S_{ent}}{\partial E_{ent}} \frac{\partial(e^{V(r)} m)}{\partial r} = 0, \quad (11)$$

where $e^{V(r)}$ denotes the gravitational redshift. Then, this equation leads to

$$F_{ent} = k_B T_{ent} \partial_r S_{ent} = -m e^{V(r)} \partial_r V(r), \quad (12)$$

where we have used Eq. (7). In the weak gravity limit $GM \ll r$, $V \simeq GM/r$, $e^V \simeq 1$, and

$$F_{ent} \simeq -m\partial_r V(r) = -\frac{GMm}{r^2}, \quad (13)$$

which is just the Newton's gravity as Verlinde showed with the thermal entropy instead of S_{ent} . Since we used the gravitational redshift for the derivation, the appearance of the Newton's gravity is not so surprising. What we want to see here is the relation between gravity felt by the test particle and quantum entanglement of the whole system. The test particle moves in a way that the total entanglement of the system maximizes.

IV. ENTANGLEMENT AND DARK ENERGY

In [11], we suggested that a cosmic causal horizon with a radius $R_h \sim O(H^{-1})$ has a kind of thermal energy $E_h \sim T_h S_h \propto R_h$, and this energy is the dark energy. Here H is the Hubble parameter, T_h is the horizon temperature, and S_h is its entropy. To be specific, we considered the entanglement energy E_{ent} associated with the cosmic event horizon for E_h . (Similar suggestions based on the Verlinde's idea [27–30] appeared recently.)

In this chapter we will redo the calculation in [11] except that we integrate Eq. (7) on the horizon surface instead of the radial direction. We will see that this gives a factor 2 smaller E_{ent} than that in [11].

As before, by integrating dE_{ent} on the surface of the event horizon we obtain

$$E_{ent} = \int_{\Sigma} dE_{ent} = k_B T_{ent} \int_{\Sigma} dS_{ent} = \frac{\hbar c}{2\pi R_h} \frac{\alpha R_h^2}{l_P^2} = \frac{c^4 \alpha R_h}{2\pi G}, \quad (14)$$

where we chose $T_{ent} = \hbar c / 2\pi k_B R_h$, the Hawking-Gibbons temperature of the horizon. Using Eq. (8) we find that this dark energy corresponds to a *constant* quantum entanglement force

$$F_{ent} = \frac{dE_{ent}}{dR_h} = \frac{c^4 \alpha}{2\pi G}, \quad (15)$$

which makes the cosmological horizon expand. (A similar value for the entropic force was obtained independently by Easson et al [30] using a surface action.) Thus, we can say that dark energy is an effective force of the universe associated with the increase of the quantum entanglement of the universe, or the radius of the cosmic causal horizon. Now, the entanglement energy density of the event horizon is given by

$$\rho_{\Lambda} = \frac{3E_{ent}}{4\pi R_h^3} = \frac{3c^4 \alpha}{8\pi^2 G R_h^2} = \frac{3c^3 \alpha M_P^2}{\pi \hbar R_h^2} \equiv \frac{3d^2 c^3 M_P^2}{\hbar R_h^2}, \quad (16)$$

which has the form of the holographic dark energy [31]. From the above equation we immediately obtain a formula for the constant

$$d = \sqrt{\frac{\alpha}{\pi}}. \quad (17)$$

If S_{ent} saturates the Bekenstein bound, then $\alpha = \pi$, hence $d = 1$. The constant d determining the equation of state w_{Λ} of the dark energy and the final fate of the universe, had been constrained only by observations before our work in [11]. Theoretically, the value $d = 1$ is favored, because it reproduces the de Sitter universe when the dark energy dominates.

One can compare predictions of our theory directly with current observational data. The equation of state for holographic dark energy is given by [31, 32]

$$w_0 = -\frac{1}{3} \left(1 + \frac{2\sqrt{\Omega_{\Lambda}^0}}{d} \right), \quad (18)$$

and its change rate at the present is [31, 33]

$$w_1 = \frac{\sqrt{\Omega_{\Lambda}^0} (1 - \Omega_{\Lambda}^0)}{3d} \left(1 + \frac{2\sqrt{\Omega_{\Lambda}^0}}{d} \right) \quad (19)$$

where z is the redshift parameter, Ω_Λ^0 is the present value of the density parameter for the dark energy, $w_\Lambda(z) \simeq w_0 + w_1(1 - R)$ and R is the scale factor of the universe. For $\Omega_\Lambda^0 = 0.73$ and $d = 1$ these equations give $w_0 = -0.903$ and $w_1 = 0.208$. These theoretical values for ω_Λ are comparable to the current observational data. Although the cosmological constant is most favored by observations, there is still a large range of values allowed by the data for the time dependent w_Λ . For example, the combination of WMAP(Wilkinson Microwave Anisotropy Probe) 7-year data, the baryon acoustic oscillation, Type Ia supernovae, and the Hubble constant yields $\omega_0 = -0.93 \pm 0.13$ and $w_1 = -0.41_{-0.71}^{+0.72}$ at the 68% *CL* [34]. (Note that the observational uncertainty for w_1 is still large, and w_1 was 0.11 ± 0.7 in WMAP-5 year data [35].)

Alternatively, we can rely on the quantum field theory to avoid the use of the Bekenstein bound. If we use the approximation $\beta_j \simeq 0.3$ for all j and use $\sum_j N_j = 118$ of the standard model of particle physics in Eq. (4), we obtain $d = 0.67$, $w_0 \simeq -1.18$ and $w_1 \simeq 0.407$. For the minimal supersymmetric standard model, $\sum_j N_j = 244$, $d = 0.962$, $\omega_0 \simeq -0.925$ and $\omega_1 \simeq 0.22$. Thus, our theory with quantum field theory is still in good agreement with the observational data, and favors supersymmetric theories than non-supersymmetric ones.

V. DISCUSSIONS

In this work, we have tried to reconcile Verlinde's theory with our theory based on quantum entanglement. There are many similarities between two theories. If we identify the horizon entropy as the entanglement entropy and the equipartition energy as the entanglement energy, we can give the theory a better ground.

We conjectured that quantum entanglement of matter and fields in the universe tend to increase over time, and there is an effective force associated with this tendency. This force might be very general in the nature and we expect someday one may measure this force during a quantum information experiments using quantum optics or solid state quantum devices.

It is also suggested that dark energy and, more fundamentally, gravity itself are quantum entanglement forces, similar to Verlinde's entropic force. If the entanglement entropy of the universe saturates the Bekenstein bound, this gives the holographic dark energy with equation of state consistent with current observational data. Our quantum informational interpretation on gravity may provide some new insights on the nature of gravity, dark energy, and arrow of time.

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