Title: Investigation into the effect of differing type of inviscid liquids in a cylindrical shell on its rotational motion along a ramp

Research Question: To what extent does the type of fluid within a hollow cylindrical can affect its dynamics while rolling down an inclined plane

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Abstract

This essay aims to explore the rotational motion of a cylindrical body along an inclined plane in order to pose an answer to the primary research question: "To what extent does the amount of fluid within a hollow cylindrical can affect its dynamics while rolling down an inclined plane?" For the sake of simplifying the understanding of the investigation, the mass of the cylindrical can will be assumed to be a negligible quantity (as will be experimentally observed) and that rotates without slipping. The reason for this is that the investigation focuses on analyzing the rotational motion of a hollow cylindrical shell with different types of liquids, and thus, frictional forces provide an additional variable to be accounted for that will not alter the final conclusions of the experimental results if it is omitted. The explanation for this resides in the fact that in the case of rolling without slipping, where the velocity at the contact point is zero, there exists a static friction force that prevents the rotating object from slipping; however, it can be assumed that this value for the friction will be similar for the various liquids and thus it does not need to be added as a factor in the investigation. Additionally, the different liquids will be assumed to be a solid cylinder (uniform mass distribution) that slides without the added factor of a frictional force within the can as well as the axis of the can consistently being assumed to be horizontal and perpendicular to the gradient of the incline. Therefore, the assumed solid cylinder will slide with negligible friction along the inclined ramp.

An experiment is conducted in order to determine a theoretically accurate and precise relationship between the type of liquid within a can and the dynamics of its rotational motion along an inclined plane through the utilization of numerical analysis. In order to do so, a cylindrical can will be fully-filled with varying liquids with the purpose of recording and analyzing the observed rotational motion. Thereafter, the time taken to reach the bottom of the

ramp is recorded and in order to establish a conclusion concerning the relationship between the density of the liquid and the time at the bottom of the ramp.

(last paragraph about experimental results)

Table of Contents

Introduction

In order to investigate the rotational motion of a hollow cylindrical can along an inclined plane, an analysis and thorough understanding of its fluid dynamics must be considered. For many years, scientists and engineers have been fascinated by the way objects roll down inclined planes, as it has many practical uses such as in transportation and industry. Of particular interest is analyzing the dynamics of the rotational motion when varying the liquid in the cylindrical shell as it poses a distinct challenge associated with accurately isolating and assessing the influence of varying the type of fluid in such a way that it retains scientific validity.

The experimental investigation into the influence of varying the type of fluid within a hollow cylindrical can on its rotational dynamics while rolling down an inclined plane holds significant implications in real-world applications across diverse fields. This research is not only a theoretical exercise but also offers valuable insights with practical relevance. For example, industries involved in food and beverage packaging are concerned with product preservation, transportation, and consumer experience. The experiment's findings can influence the choice of fluids for packaging liquids. Consider the rotational behavior of cylindrical containers filled with beverages during transportation. Understanding how various fluids affect rotation can help manufacturers select fluids that minimize agitation, preserving product quality and preventing spills.

The exploration into the effects of various fluids within a hollow cylindrical can on its rotational behavior during its descent down an inclined plane yields insights with far-reaching practical implications. This experiment not only delves into fundamental scientific principles but also unveils applications across diverse industries. A notable facet of this investigation involves treating the liquids within the can as solid cylinders, a simplification that allows for streamlined calculations of the moment of inertia. It's important to note that this assumption is made based on the specific context of the experiment, and its validity hinges on the properties of the liquids under consideration. The reason that the properties of the chosen fluids are put into consideration as the investigation is focusing on pure liquids that are uniform in density is because if the density is not uniform the relationship between density would have to be understood in terms of the sloshing dynamics and internal friction of the liquid within the hollow cylindrical can which would be outside of the scope of this investigation. Moreover, the hollow cylindrical shells will be fully-filled in such a way that the center of mass for an equivalent solid cylindrical shell would be equal.

In engineering, the experiment's examination of how different fluids impact rotational dynamics holds the promise of revolutionizing the optimization of machinery and devices. Industries ranging from mechanical engineering to energy production stand to benefit significantly from the insights gained. By delving into the nuanced relationship between fluid properties and rotational behavior, engineers can reshape the design and operation of mechanical systems. This spans from enhancing the efficiency and durability of rotating components within industrial machinery to revolutionizing energy production through the meticulous selection of fluids that streamline rotational motion in turbines and generators. Moreover, this understanding can drive innovations in automotive engineering by facilitating the development of transmission fluids that optimize rotational dynamics, resulting in smoother shifts, reduced heat generation, and improved fuel efficiency. Ultimately, the experiment's findings resonate across engineering disciplines, paving the way for the refinement and advancement of crucial machinery and devices.

Theoretical Background

Cylindrical shell rotational motion along an inclined plane

In the context of a hollow cylinder fully–filled with water rolling down an inclined plane where friction is not negligible, its translational motion may be described through the use of Newton's second law as can be seen in Eqs. 1 and 2 and illustrated in Figure 2.

$$\sum F = ma \ (1)$$

$$mgsin\theta - f = ma \ (2)$$

where the variables are defined with m being the mass of the cylinder (can), g is the gravitational acceleration of the earth, a is the linear acceleration of the motion of the cylinder, θ is the angle of inclination of the path, and f is the frictional force that exists between the ramp and the cylinder.

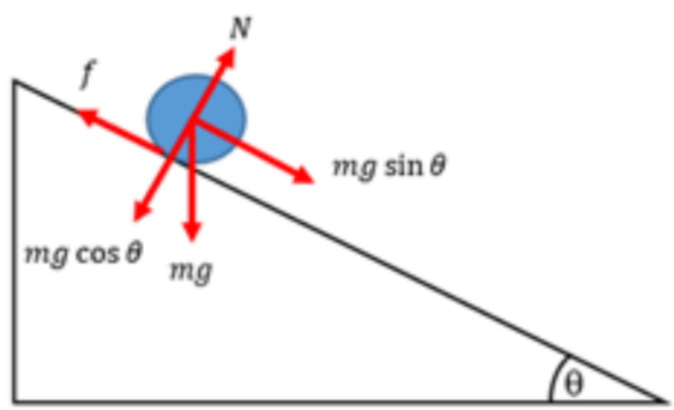

Figure 2. Illustration of the forces acting on the center of mass of the fluid-filled hollow cylindrical shell

The primary source of rotational motion is a result of a quantity called the moment of force or torque that is often denoted by τ which produces the rotational acceleration of a rotating object. Therefore, in regards to a rotational motion to the center of mass of a cylinder, the τ which has a direction that is stated to be perpendicular to the axis of rotation is expressed in Eq. 3.

$$\tau = rf \ (3)$$

Furthermore, Newton's second law of rotational motion is expressed as seen in Eq. 4.

$$\tau = I\alpha \ (4)$$

The angular acceleration (α) in a situation where there is an object experiencing rotational motion with radius r is expressed as shown in Eq. 5.

$$\alpha = \frac{a}{r} \ (5)$$

The moment of inertia of an object experiencing rotational motion with mass m and radius r is often expressed as shown in Eq. 6.

$$I = \frac{1}{2}mr^2 \ (6)$$

where k is known to be the moment of inertia coefficient for a cylinder which is equal to $\frac{1}{2}$. Therefore, through substituting the relationships revealed through Eqs. 5 and 6 into Eq. 4 to then connect it with Eq. 3, we can express a new Eq. 7.

$$f = \frac{1}{2}ma \ (7)$$

Furthermore, if we substitute this value for f from Eq. 7 into Eq. 2, then we obtain Eq. 8.

$$a = \frac{2}{3}gsin\theta \ (8)$$

In order to further simplify the equation, we can express the acceleration as the change in velocity with respect to time as shown in Eq. 9.

$$\frac{dv}{dt} = \frac{2}{3}gsin\theta \ (9)$$

Moreover, according to a derivation further made by Ahmad Kassim through the use of the moment of inertia of a hollow cylinder, which is $I = mr^2$, the acceleration is expressed as:

$$a = \frac{1}{2}gsin\theta \ (10)$$

It must be noted that these equations only apply to cylindrical shells that are assumed to be either fully solid due to the fact that the shell has been fully-filled with water; therefore, they can be treated as a simple cylinder with a known moment of inertia. Furthermore, in the context of the experiment which will be performed, Eq. 9 and Eq. 10 will apply to all situations presented because, as discussed before, the fluids in the can will be treated as a solid cylinder.

Conservation of energy analysis

However, a different approach which utilizes the law of conservation of energy may be taken in order to investigate the dynamics associated with the particular case of rotational motion that arises when the cylindrical shell is only partially-filled. According to the law of conservation of mechanical energy along an inclined plane, an object that satisfies the conditions of having an initial velocity equal to zero can be expressed through the following equation:

$$\frac{1}{2}mv^2 = mgh \ (11)$$

where m is the mass of the object, v is the final velocity of the object, and h is the initial height of the object at its position of rest. The object's initial height, h, can be expressed by the following:

$$h = lsin\theta \ (12)$$

where l is the length of the object's path. Moreover, through substituting Eq. 10 into Eq. 11, a new Eq. 12 can be expressed:

$$\frac{1}{2}v^2 = glsin\theta \ (13)$$

Therefore, as it is a universally accepted fact that the acceleration of an object moving along an inclined plane is constant; the kinematic equation that expresses the acceleration of an object moving from rest with a constant acceleration may be described as seen in Eq. 13:

$$al = \frac{v^2}{2} \quad (14)$$

Therefore, through simple algebra after taking into account Eq.'s 12 and 13, Eq. 14 can be found to be:

$$a = gsin\theta \ (15)$$

$$\frac{dv}{dt} = gsin\theta \ (16)$$

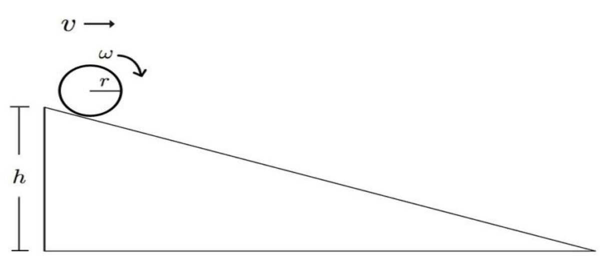

Figure 3. Illustration of the initial position of the cylindrical shell and its intended trajectory along the inclined plane

Furthermore, since the energy of the system must be conserved, the potential energy that the cylindrical shell initially experiences at the top of the inclined plane which will be denoted with the height, h, will be equal to the accumulated kinetic energy at the end of the ramp that includes both the translational and rotational kinetic energies. If the potential energy (E_p), the translational kinetic energy ($E_{k(T)}$), and the rotational kinetic energy ($E_{k(R)}$) are expressed respectively as:

$$E_p = mgh \; (17)$$

$$E_{k(T)} = \frac{1}{2}mv^2 \; (18)$$

$$E_{k(R)} = \frac{1}{2}I\omega^2 \; (19)$$

Then, the conservation of energy in this scenario is represented by:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \; (20)$$

Furthermore, it is known that the moment of inertia, I, of a solid cylinder is $I = \frac{1}{2}mr^2$, and on the other hand, the moment of inertia of a completely hollow cylinder is $I = mr^2$. Therefore, through further stating that it is known that angular velocity, ω, has a direct relationship with translational velocity through $\omega = \frac{v}{r}$ then the final velocities of each object can be analyzed. In regards to the hollow cylinder, the equation of conservation of mechanical energy may be expressed as:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \; (21a)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mr^2\frac{v^2}{r^2} \; (21b)$$

$$mgh = mv^2 \; (21c)$$

$$\therefore v = \sqrt{gh} \; (21d)$$

Moreover, in regards to the solid cylinder, the equation of the conservation of mechanical energy may similarly be expressed as:

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2} \ (22a)$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{4}mr^2\frac{v^2}{r^2} \ (22b)$$

$$mgh = \frac{3}{4}mv^2 \ (22c)$$

$$\therefore v = \sqrt{\left(\frac{4gh}{3}\right)} \ (22d)$$

Therefore, through simple mathematical observation, the final velocity of the solid cylinder will be greater than that of the hollow cylinder. Furthermore, it can be stated that an object's final velocity in this scenario is directly related to its moment of inertia rather than its mass. As a result of this observation, it is paramount to the evaluatory aspect of the investigation to put into consideration the differences in moment of inertia in order to discuss the dynamics of a cylindrical shell along an inclined plane as less dense fluids will act more similarly to a rotating hollow cylinder while the denser fluids will act more similarly to a rotating cylindrical solid.

Methodology

Variables

Table 1: Independent and Dependent Variables

Variable	**Significance**	**How It Was Kept Constant**
Type of Fluid	Investigating how different fluids impact rotational dynamics	The experiment utilized a hollow cylindrical can with constant dimensions and mass. Different types of fluids were used to fill the can, altering only the fluid while keeping the can properties consistent.

Inclined Plane Angle	Determining how the angle affects rotational behavior	The angle of the inclined plane (e.g., 10.03 ± 2 degrees) was maintained consistently for all trials, ensuring that the gravitational force component remains the same across experiments.
Amount of Fluid	Analyzing the effect of fluid quantity on rotation	To keep this variable constant, the cylindrical can was fully filled with each fluid during every trial, eliminating variations in the mass distribution within the can.
Initial Release Point	Studying the initial conditions of motion	The can was released from the same point at the top of the incline for all trials, controlling the initial potential energy and velocity at the beginning of each experiment.

Table 2: Variables Kept Constant

Variable	**Significance of Keeping It Constant**	**How It Was Kept Constant**
Can Geometry and Mass	Isolating the influence of fluid type by keeping the can's properties consistent	Using a uniform cylindrical can with identical dimensions and mass across all trials, ensuring that variations in rotation are solely attributed to fluid properties.
Inclined Plane Angle	Eliminating variations due to different angles of inclination	Maintaining a fixed angle of inclination (e.g., 10.03 ± 2 degrees) for all trials to ensure consistent gravitational force contributions that impact the can's motion.
Initial Release Point	Controlling the initial conditions to ensure consistent starting conditions	Releasing the can from the same point at the top of the incline for each trial, standardizing the initial potential energy and velocity.

Apparatus

a. Wooden Ramp (1.454 ± 0.001 m)

c. 30cm Ruler (for precise measurements)

d. Stopwatch (for timing the can's descent)

e. Fluids: Sunflower Cooking Oil, Automated Transmission Fluid, Water, and Honey (for experimental trials with different fluids)

f. Cylindrical Hollow Can (for the experiment's main object of study)

g. 1kg hexagonal weight (2)

h. Duct tape (1)

i. Digital mass scale (1)

Procedure

Setup and Calibration:

- Set up the wooden ramp on a stable surface, ensuring it is secure and stationary.
- Use a leveling tool to ensure the ramp is properly aligned and has a consistent angle of inclination (e.g., 10.03 ± 2 degrees). Adjust if needed.
- Place the cylindrical hollow can at the top of the ramp, aligning it with the center of the ramp and perpendicular to the surface.

Fluid Preparation:

- Prepare the four fluids: sunflower cooking oil, automated transmission fluid, water, and honey.
- Ensure each fluid is at room temperature to eliminate temperature variations affecting the experiment.

Trial Execution:

- Begin with the can filled with one type of fluid (e.g., sunflower cooking oil).
- Measure and record the can's dimensions (radius and height) and mass.
- Hold the can at the top of the ramp, ensuring it is still and not rotating.
- Release the can gently from rest, allowing it to roll down the ramp without any additional force.
- Start the stopwatch as soon as the can is released and stop it when the can reaches a designated point on the ramp. Use a visual marker or a distinct feature on the ramp to mark the point.
- Repeat this process six times for each fluid type.

Data Collection:

- Repeat the trial six times for each fluid type to ensure accuracy and consistency.

- Empty and thoroughly clean the can between trials, removing any residual fluid and ensuring no cross-contamination.
- Repeat the entire procedure for the remaining fluid types: automated transmission fluid, water, and honey.

Data Analysis:

- Calculate the average time taken for the can to travel the specified distance for each fluid type.
- Analyze the collected data to identify trends and patterns in the can's rotational dynamics for different fluids.
- Compare the average times and infer any differences in rotational behavior.

Observations and Interpretations:

- Record any observations related to the can's motion, such as differences in speed, smoothness, or any visible effects on rotation.
- Interpret the observed results based on the principles of rotational motion and fluid dynamics.

Raw data tables

Table 3: Dimensions of Hollow Cylindrical Can

Dimensions of can	**Trial 1**	**Trial 2**	**Trial 3**
Radius ± 0.0005 (m)	3.5	3.45	3.48
Height (m) ±0.0005	11.3	11.28	11.35

Table 4: Mass of Each Liquid With and Without the Duct Tape Covering (where mass of can is negligible)

Fluid Type	**Trials**	**Mass of liquid + tape**	**Mass of liquid (kg)**
Automated transmission fluid dextrin			
	Trial 1	334	333
	Trial 2	334	333
	Trial 3	334	333
Sunflower oil			
	Trial 1	414	413
	Trial 2	414	413
	Trial 3	414	413
Water			
	Trial 1	445	444
	Trial 2	445	444
	Trial 3	445	444
Honey			
	Trial 1	550	549
	Trial 2	550	549
	Trial 3	550	549

Table 5: Time taken to reach bottom of ramp in seconds for each liquid fully-filled into a hollow cylindrical shell.

Fluid Type	**Trials**	**Time taken to reach bottom of ramp** ± 0.4 **(s)**
Automated transmission fluid dextrin		
	Trial 1	1.98
	Trial 2	2.01
	Trial 3	2.02
Sunflower oil		
	Trial 1	1.70
	Trial 2	1.72
	Trial 3	1.75
Water		
	Trial 1	1.27
	Trial 2	1.28
	Trial 3	1.25
Honey		
	Trial 1	1.98
	Trial 2	2.01
	Trial 3	2.02

Table 6: Measurements of the diagonal length (hypotenuse) and height of the wooden ramp in meters

Trials	Hypotenuse (diagonal length	Height (m)

	of ramp, m)	
Trial 1	145.4	
Trial 2	144.8	
Trial 3	144.5	

<u>Processed data tables</u>

Table 7:

Average height ± 0.0015 (m)	Average radius ± 0.0015 (m)	Average volume of the can ± 0.0060 (m^3)
11.31	3.47	427.83

Table 8: Angle of in