Homework Problem #4

Potential step

Consider a potential step with a height of $V_0 = 0.5$ eV. An electron is incident from the lower potential side (where the potential energy is set equal to zero) with an energy $E = 0.3$ eV.

Find the penetration depth which is defined as the distance into the barrier region at which the probability decreases to $1/e$ of its initial value at the potential discontinuity, $z = 0$

Give your answer in unit of nm. Answers within 5% error will be considered as correct.

Ans: The penetration distance η in a finite potential well is the distance at which the wave function has decreased to $1/e$ of the wave function at the classical turning point:

$$\psi(x = L + \eta) = \frac{1}{e}\psi(L)$$

The penetration distance can be shown to be

$$\eta = \frac{\hbar}{\sqrt{2m(V_0 - E)}}$$

The probability of finding the particle beyond the penetration distance is nearly zero.

Given, $V_0 = 0.5\,eV$

$= 0.5 \times 1.6 \times 10^{-19}\,J$

$E = 0.3\,eV = 0.3 \times 1.6 \times 10^{-19}\,J$

$V_0 - E = 0.2\,eV = 0.2 \times 1.6 \times 10^{-19}\,J$

$m_e = 9.11 \times 10^{-31}\,kg$

$\hbar = 1.05 \times 10^{-34}\,J.s$

(3)

$$\eta = \frac{1.05 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 0.2 \times 1.6 \times 10^{-19}}}$$

from calculator

$$\eta = 4.34 \times 10^{-10}$$

$$\eta = 0.000000000434$$

or

$$\eta = 0.434 \times 10^{-9} m$$

$$\boxed{\eta = 0.434 nm}$$