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Spontaneous Emission *vs.* Vacuum Fluctuations (*).

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Summary. — Spontaneous emission exists both in classical and quantum theories. Vacuum fluctuations of the electromagnetic field give an essential contribution to the intensity of the spontaneous emission in the low-lying levels of matter. Vacuum fluctuations play a crucialrole when matter is in its ground state. They are the cause of the stability of the ground state.

1. – The question about the relation between spontaneous emission and vacuum fluctuations has a long history. Enough to mention that as early as 1935 WEISSKOPF (1) claimed that spontaneous emission is ascribed entirely to the zero-point fluctuations of the electromagnetic field. Since that time this statement was criticized on various occasions. In 1939 GINZBURG (2) showed that spontaneous emission is not purely a quantum phenomenum (as it has to be if it is due to the vacuum fluctuations) and exists in classical theory as well.

This statement is almost trivial if we recall the phenomenon of the radiation damping of the classical oscillator. The role of vacuum fluctuations in

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

(1) V. WEISSKOPF: *Naturwissenschaften*, **23**, 631 (1935).

(2) V. L. GINZBURG: *Dokl. Akad. Nauk SSSR*, **24**, 130 (1939).

the spontaneous emission was analysed by the present author ([3-5]). Quite recently the interest in this problem has been renewed (see, *e.g.*, ([6-8])).

Despite the long history of the question, it seems that until now there is a lack of clear understanding regarding the relation between spontaneous emission and vacuum fluctuations. In his paper ([8]), ROSE provides the result of an informal canvass of his colleagues concerning spontaneous emission. The majority of them was convinced that the zero-point fluctuations « induce » spontaneous emission. I have repeated his « experiment » and obtained the same result. ROSE also mentioned that in popular testbooks of Eisberg and Resnik ([9]), Schiff ([10]) and Baym ([11]) the same point of view had been expressed.

All this motivated me to return again to the problem I have once analysed.

2. – Our aim is to compare classical and quantum calculations of the intensity of spontaneous emission and to « pick up » the specific role of zero-point fluctuations of the field.

For a better understanding, let us begin with a simple example of harmonic-oscillator spontaneous radiation.

We assume that, at the initial time $t = 0$, the quantum harmonic oscillator was in a state with definite energy (or in a mixture of such states) and the radiation field was in the vacuum state, *i.e.* with all numbers of photons equalling zero.

If we wish to give the exact analog of such a situation in the classical theory, we should deal with an ensemble of classical oscillators with entirely random phases. The calculation of average meanings in such an ensemble would then correspond to quantum averaging ([4]). For example, the average meaning of the co-ordinate q of the quantum harmonic oscillator with energy levels $E_n = (n + \frac{1}{2})\hbar\omega$ even in the classical region $n \to \infty$ does not turn out into the classical solution $q = q_0 \cos(\omega_0 t + \theta)$. The mean value of the co-ordinate is equal to zero in this case, as should be for the average in the ensemble of classical oscillators with random phases θ. Of course, the same relates to the radiation oscillators.

The result of the classical calculation of the intensity of radiation in one

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([3]) B. FAIN: *Izv. Vyssh. Uchebn. Zaved. Radiofiz.*, **6**, 207 (1963).
([4]) B. FAIN and YA. I. KHANIN: *Quantum Electronics*, Vol. **1** (Oxford, 1969).
([5]) B. FAIN: *Photons and Nonlinear Media* (Moscow, 1972) (in Russian).
([6]) *Foundations of Radiation Theory and Quantum Electrodynamics*, edited by A. O. BARUT (New York, N. Y., 1980).
([7]) A. J. DEGREGORIA: *Nuovo Cimento A*, **51**, 377 (1979).
([8]) A. ROSE: *Phys. Status Solidi A*, **61**, 133 (1980).
([9]) R. EISBERG and R. RESNICK: *Quantum Physics* (New York, N. Y., 1974).
([10]) L. I. SCHIFF: *Quantum Mechanics*, 3rd ed. (New York, N. Y., 1955).
([11]) G. BAYM: *Lectures on Quantum Mechanics* (Reading, Mass., 1973).

mode ν (in the dipole approximation) has the form ((4), p. 212)

$$\text{(1)} \qquad \frac{\mathrm{d}\overline{H}_\nu}{\mathrm{d}t} = \frac{e^2\pi}{2mc^2} A^2_{\nu x} \left\{ \left[\frac{\overline{p_x^2(0)}}{2m} + \frac{m\omega_0^2\overline{q^2(0)}}{2} \right] - \frac{1}{2}[\overline{p_\nu^2(0)} + \omega_\nu^2\overline{q_\nu^2(0)}] \right\} \delta(\omega_0 - \omega_\nu)\,,$$

where $A_{\nu x}$ is the projection of the normal mode of the vector potential $\boldsymbol{A}_\nu$ on the direction x of the harmonic oscillator (for standing waves in free space $|\boldsymbol{A}_\nu|$ is equal to $\sqrt{8\pi/L^3}\,c$, L^3 being the volume of the space, $L\to\infty$), p_ν, q_ν are the momenta and co-ordinates of radiation oscillators. All other notations are obvious.

The same formula (1) may be derived in quantum theory (4), the only difference being that averaging in (1) is performed in quantum ensembles.

Now, in the classical theory, to calculate the intensity of spontaneous radiation, one should put $\frac{1}{2}[\overline{p_\nu^2(0)} + \omega_\nu^2\overline{q_\nu^2(0)}] = 0$ and sum over all radiation modes of free space. Such a summation (as a matter of fact the integration over the continuum of modes) would lead to the well-known classical expression of the intensity of radiation

$$\text{(2)} \qquad I = \frac{2}{3c^2}\overline{(\ddot{\boldsymbol{d}})^2}\,,$$

where $d = eq$ is the dipole moment of the oscillator.

The same result we would obtain from (1) in the so-called semi-classical theory when *treating the oscillator quantum-mechanically* and the *radiation field classically*, the only difference being that averaging in (2) means averaging over the quantum ensemble.

Here we want to stress that *such a semi-classical approach which treats the matter quantum-mechanically and the field classically is, generally speaking, controversial and not self-consistent.* It gives a correct expression in the classical region (*i.e.* at highly excited levels of the oscillator), but leads to wrong implications at lower levels of the quantum oscillator (which is now treated in a quantum way). Thus, in the ground state of the quantum oscillator,

$$\frac{\overline{p_x^2(0)}}{2m} + \frac{m\omega_0^2\overline{q^2(0)}}{2} = \frac{1}{2}\hbar\omega_0 \quad \text{and} \quad \overline{(\ddot{\boldsymbol{d}})^2} \neq 0\,.$$

It means that the oscillator radiates even in its ground state. Of course, such a conclusion contradicts the experimentally observed stability of matter.

Now, if we treat both the oscillator and the radiation according to the quantum theory, we should take into account that the absence of radiation at the initial moment means that all numbers of photons are zero, but zero-point fluctuations of the field are present:

$$\frac{1}{2}\left(\overline{p_\nu^2(0)} + \omega_\nu^2 \overline{q_\nu^2(0)}\right) = \frac{1}{2}\hbar\omega_\nu . \tag{3}$$

In particular, it means that, if the oscillator is initially in the ground state, then the zero-point fluctuations exactly compensate the emission connected with the zero-point fluctuation of the oscillator (see eq. (1)).

Thus we come to the quite obvious conclusion that, in order to achieve noncontroversial results for the spontaneous emission, we must treat both the matter and the field according to the quantum theory. Another conclusion which stems from the example of the harmonic oscillator interacting with the radiation is that nonspontaneous emission is purely a quantum effect owing to the vacuum fluctuations, but *the absence of this emission in the ground state is purely a quantum effect which is due to the vacuum fluctuations.*

Of course, everything said above is typical not only of the harmonic oscillator, but of every system.

It has been shown in ref. ([3-5]) that the intensity of the spontaneous radiation of a two-level system may be presented by the formula

$$I_\nu = I_\nu^0\left[\frac{1}{2} + (n_+ - n_-)\frac{\overline{p_\nu^2(0)} + \omega_\nu^2\overline{q_\nu^2(0)}}{2\hbar\omega_\nu}\right], \tag{4}$$

where I_ν^0 is the intensity of the spontaneous emission of the two-level system in its upper level, $n_\pm$ are the mean occupation numbers in levels $+$ and $-$ and $n_+ + n_- = 1$. Thus, when the two-level system is at its upper level $+$, $n_+ = 1$, $n_- = 0$ and the field is in the vacuum state (3), the vacuum fluctuation contributes half of the total radiation, the other half is connected with fluctuations of the dipole moment and may be obtained semi-classically (the two-level system is considered quantum-mechanically and the field classically). This explains the factor $\frac{1}{2}$ that appears in the usual formulation of the correspondence principle ([12]).

When the two-level system is in its ground state ($n_- = 1$, $n_+ = 0$), the vacuum fluctuations exactly compensate the first term on the r.h.s. of (4) and the total intensity vanishes. In the general case, the energy of the interaction between radiation and matter may be represented in the form ([4])

$$V = -\sum_\nu \hat{B}_\nu \hat{q}_\nu ,$$

where $\hat{B}_\nu$ is the operator depending on the variables of matter and $\hat{q}_\nu$ is the

([12]) W. HEITLER: *The Quantum Theory of Radiation* (Oxford, 1954).

operator of the co-ordinate of the radiation oscillator. In this case the intensity of the radiation of the ν-th mode may be represented as [3,4]

$$I_\nu = \pi(B_\nu^2)_{\omega_\nu} - \frac{1}{\omega_\nu}\chi''(\omega_\nu)\left(\bar{n}_\nu + \frac{1}{2}\right)\hbar\omega_\nu . \tag{5}$$

Here $(B_\nu^2)_{\omega_\nu}$ is the density of the steady-state fluctuations of the quantity B_ν (which is proportional to the dipole moment in the dipole approximations) and $\chi''(\omega_\nu)$ is the imaginary part of the susceptibility defined as a characteristic of the response of matter to the classical field:

$$q_\nu = \mathrm{Re}\, q_\nu^0 \exp[-i\omega_\nu t] , \qquad \langle B_\nu \rangle = \mathrm{Re}\,\{\chi(\omega_\nu)\, q_\nu^0 \exp[-i\omega_\nu t]\} .$$

Again the first term on the r.h.s. corresponds to the radiation calculated in the semi-classical approach (the radiation field is treated classically) and the second term corresponds to the vacuum fluctuations (as is clear from (5)). It can be shown that in the ground state

$$(B_\nu^2)_{\omega_\nu} = \frac{\hbar\chi''(\omega_\nu)}{\pi}$$

and $I_\nu^s = 0$, *i.e.* the ground state does not radiate.

3. – From the above several conclusions follow.

1) Spontaneous emission exists both in classical and in quantum theories.

2) The semi-classical approach, when matter is treated quantum-mechanically and radiation in the classical way, does not give correct results at lowlying levels of matter.

3) The crucial role of the vacuum fluctuations emerges in the ground state of matter. *The stability of the ground state* (*i.e.* the fact that it does not radiate) *is purely a quantum effect which is due to the vacuum fluctuations.*

● RIASSUNTO (*)

Emissione spontanea esiste nelle teorie sia classiche che quantistiche. Le fluttuazioni nel vuoto del campo elettromagnetico danno un contributo essenziale all'intensità dell'emissione spontanea nei livelli inferiori della materia. Le fluttuazioni nel vuoto giocano un ruolo cruciale quando la materia è nel suo stato fondamentale. Esse sono la causa della stabilità dello stato fondamentale.

(*) *Traduzione a cura della Redazione.*

Спонтанное излучение в зависимости от флуктуаций вакуума.

Резюме (*). — Спонтанное излучение существует и в классической и в квантовой теориях. Вакуумные флуктуации электромагнитного поля дают существенный вклад в интенсивность спонтанного излучения на низколежащих уровнях вещества. Флуктуации вакуума играют существенную роль, когда вещество находится в основном состоянии. Эти флуктуации обуславливают устойчивость основного состояния.

(*) *Переведено редакцией.*