second displacement $\vec{B}$: $\quad B_y = (4\text{ km})\sin 60° = 3.46\text{ km}$

3. The components of the resultant displacement $\vec{C} = \vec{A} + \vec{B}$ are found by addition:

$$C_x = A_x + B_x = -3\text{ km} + 2\text{ km} = -1\text{ km}$$

$$C_y = A_y + B_y = 0 + 3.46\text{ km} = 3.46\text{ km}$$

4. The Pythagorean theorem gives the magnitude of $\vec{C}$:

$$C^2 = C_x^2 + C_y^2 = (-1\text{ km})^2 + (3.46\text{ km})^2 = 13.0\text{ km}^2$$

$$C = \sqrt{13.0\text{ km}^2} = 3.61\text{ km}$$

5. The ratio of C_y to C_x gives the tangent of the angle θ between $\vec{C}$ and the x axis:

$$\tan\theta = \frac{C_y}{C_x} = \frac{3.46\text{ km}}{-1\text{ km}} = -3.46$$

$$\theta = \tan^{-1} -3.46 = -74°$$

Remarks Since the displacement (which is a vector) was asked for, the answer must include either the magnitude *and* direction, or both components. In (*b*) we could have stopped at step 3 because the x and y components completely define the displacement vector. We converted to the magnitude and direction to compare with the answer to part (*a*). Note that in step 5 of (*b*), a calculator gives the angle as $-74°$. But the calculator can't distinguish whether the x or y component is negative. We noted on the figure that the resultant displacement makes an angle of about 75° with the negative x axis and an angle of about 105° with the positive x axis. This agrees with the results in (*a*) within the accuracy of our measurement.

Unit Vectors

A **unit vector** is a *dimensionless* vector with unit magnitude. The vector $A^{-1}\vec{A}$ is an example of a unit vector that points in the direction of $\vec{A}$. Unit vectors are often written boldface italic with an overhead caret as in $\hat{A} = A^{-1}\vec{A}$. Unit vectors that point in the x, y, and z directions are convenient for expressing vectors in terms of their rectangular components. They are usually written $\hat{i}$, $\hat{j}$, and $\hat{k}$, respectively. Then the vector $A_x\hat{i}$ has a magnitude A_x and points in the positive x direction (or negative x direction if A_x is negative). A general vector $\vec{A}$ can be written as the sum of three vectors, each of which is parallel to a coordinate axis (Figure 3-12):

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \tag{3-7}$$

The addition of two vectors $\vec{A}$ and $\vec{B}$ can be written in terms of unit vectors as

$$\begin{aligned}\vec{A} + \vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) + (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\\ &= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}\end{aligned} \tag{3-8}$$

The general properties of vectors are summarized in Table 3-1.

Exercise Given two vectors

$$\vec{A} = (4\text{ m})\hat{i} + (3\text{ m})\hat{j} \quad \text{and} \quad \vec{B} = (2\text{ m})\hat{i} - (3\text{ m})\hat{j}$$

find (*a*) A, (*b*) B, (*c*) $\vec{A} + \vec{B}$, and (*d*) $\vec{A} - \vec{B}$. (*Answers* (*a*) $A = 5$ m, (*b*) $B = 3.61$ m, (*c*) $\vec{A} + \vec{B} = (6\text{ m})\hat{i}$, (*d*) $\vec{A} - \vec{B} = (2\text{ m})\hat{i} + (6\text{ m})\hat{j}$)

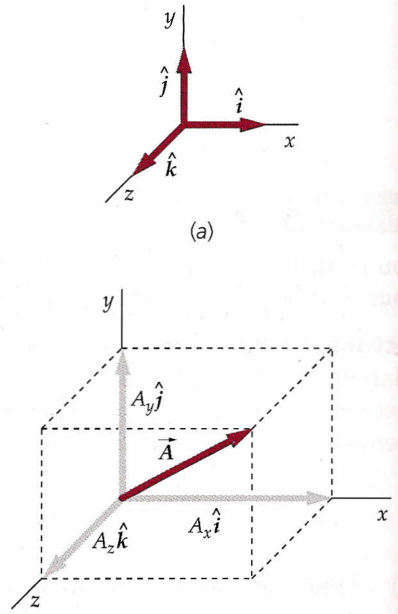

Figure 3-12 (*a*) The unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ in a rectangular coordinate system. (*b*) The vector $\vec{A}$ in terms of unit vectors: $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$.