Equations as Guides to Thinking and Problem Solving

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Science is the study of nature's rules. The most basic of these are the laws of physics, most of which are expressed in equation form. Physics equations show how concepts connect to one another. But does a study of these equations enhance student understanding? Not always, for too often in an introductory course students are tempted (or even encouraged) to memorize equations or keep them in a handy list and then, when confronted with a problem, to look for what might be a relevant equation and plug numbers into it. Little understanding results. Or worse, equations may take a backseat in an introductory course and get little or no attention at all (as some popular physics books boast *no* equations).

Equations in more advanced courses take a front seat: to exhibit, in a beautiful and concise way, relationships that exist in nature, which can be manipulated by students in order to see new relationships and derive new results.

Must this focus on equations apply only to advanced courses? My experience is that emphasis on equations is not only possible in introductory courses, but is highly desirable. Teaching equations as guides to thinking is akin to teaching notes on musical scores to guide musicians.

What do the equations of physics contain? Symbols—symbols representing concepts. The meaning of the symbols has been the focus of my conceptual physics courses over many years. My emphasis has gone beyond what the symbols represent, to what they mean—how their presence in equations shows the connections in nature. Focus on these connections can make an introductory course more enjoyable by being more meaningful and more effective.

There are two aspects to the idea of equations as guides to thinking. One is just examining and appreciating equations for what they reveal about relationships among concepts—the inverse-square dependence of gravitational and electric forces on distance, for instance, or the proportionality of impulse to momentum change. Every equation is capable of being held up, turned around, examined, and discussed, much as a painting or a musical composition might be for students of art or music.

The second aspect is the manipulation and combination of equations in search of a particular result, *before* numbers are inserted. After the independence of horizontal and vertical motion for projectiles has been discussed (an intriguing discovery for physics newcomers!), consider this problem: A horizontally moving tennis ball barely clears the net to land within the tennis court. (a) Given that the height of the ball as it clears the net is y and the distance from the bottom of the net to the edge of the court is x, show that the maximum speed of the ball is

$$\frac{x}{\sqrt{\frac{2y}{g}}} \quad \text{(or equivalently, } \sqrt{\frac{x^2 g}{2y}}\text{).}$$

Once students become familiar with the equations of kinematics and their symbols, this problem is not too difficult. What about numbers, which are a part of physics too? Only after the symbolic solution, does a part (b) ask: Given that the height of the ball clearing the net is 0.92 m and the distance to the edge of the court is 11.9 m, find the maximum speed of the ball. And perhaps a part (c) that asks: How would your answer differ if the net were slightly higher?

Students who have not been taught via symbols are intimidated by the symbolic problem above and greatly prefer addressing part (b), with no part (a). For these students, plugging and chugging is the preferred mode. And part (c) seems to be a "tricky" question. Aren't these the students who choose not to continue in physics?

On the other hand, students who have been taught the meaning of symbolic notation in physics and have examined equations for *what they mean* welcome symbolic problem solving. Instances where symbols cancel and save steps to a solution are almost automatic in symbolic problems. More important is mind-set: When problems are couched in symbols, and the numbers held for later, a student's task calls for thinking that calculators can't supply. They think concepts.

Isn't teaching emphasis on symbols and their meanings in an introductory course a worthwhile effort?

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