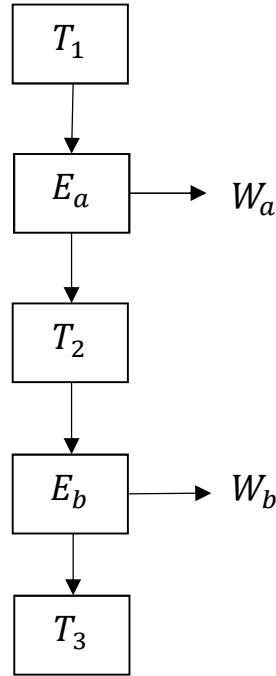


The assertion: of two Carnot engines E_a and E_b operating in series,



Where temperatures are

$$T_1 > T_2 > T_3,$$

the Carnot efficiency of this two-engine series \mathcal{C} is asserted to be

$$\eta_{\mathcal{C}} = 1 - \frac{T_3}{T_1}.$$

which would be the same as one engine running directly between T_1 and T_3 . Is this correct ??

Jaynes, in his paper The Evolution of Carnot's Principle, says (equation 4):

$$\eta_{13} = \eta_{12} + \eta_{23} - \eta_{12}\eta_{23}$$

where $\eta_{13} = \eta_{\mathcal{C}}$. How is this obtained? If we express the efficiency of the series \mathcal{C} in terms of the individual efficiencies of engines E_a and E_b

$$\eta_a = 1 - \frac{T_2}{T_1}, \quad \eta_b = 1 - \frac{T_3}{T_2}$$

$$\eta_{\mathcal{C}} = \eta_a \eta_b = \left(1 - \frac{T_2}{T_1}\right) \left(1 - \frac{T_3}{T_2}\right) \text{ (is this a justified assertion?)}$$

$$\eta_a \eta_b = 1 - \frac{T_3}{T_2} - \frac{T_2}{T_1} + \frac{T_3}{T_1} = \eta_b - \frac{T_2}{T_1} + \frac{T_3}{T_1}$$

$$\eta_a \eta_b = 1 - \frac{T_3}{T_2} - \frac{T_2}{T_1} + \frac{T_3}{T_1} = \eta_a - \frac{T_3}{T_2} + \frac{T_3}{T_1}$$

$$2\eta_a \eta_b = \eta_b - \frac{T_2}{T_1} + \frac{T_3}{T_1} + \eta_a - \frac{T_3}{T_2} + \frac{T_3}{T_1}$$

$$2\eta_a\eta_b = \eta_a + \eta_b - \frac{T_2}{T_1} + \frac{T_3}{T_1} - \frac{T_3}{T_2} + \frac{T_3}{T_1}$$

$$2\eta_a\eta_b = \eta_a + \eta_b + \eta_a\eta_b - 1 + \frac{T_3}{T_1}$$

$$\eta_a\eta_b = \eta_a + \eta_b - 1 + \frac{T_3}{T_1}$$

$$\eta_a\eta_b = \eta_a + \eta_b - \left(1 - \frac{T_3}{T_1}\right)$$

$$\eta_a\eta_b = \eta_a + \eta_b - \eta_c$$

which says that the 2-engines in series have an efficiency = the sum of the individual efficiencies minus the efficiency of one engine running between the maximum temp gradient. This seems not to be consistent with neither Jaynes nor the assertion that the 2-engine series efficiency should be $\left(1 - \frac{T_3}{T_1}\right)$.