(c) Calculate the energy of this configuration.

Problem 3.8 In Ex. 3.2 we assumed that the conducting sphere was grounded ($V = 0$). But with the addition of a second image charge, the same basic model will handle the case of a sphere at *any* potential V_0 (relative, of course, to infinity). What charge should you use, and where should you put it? Find the force of attraction between a point charge q and a *neutral* conducting sphere.

Problem 3.9 A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy plane.)

(a) Find the potential in the region above the plane.

(b) Find the charge density σ induced on the conducting plane.

Example 3.2

A point charge q is situated a distance a from the center of a grounded conducting sphere of radius R (Fig. 3.12). Find the potential outside the sphere.

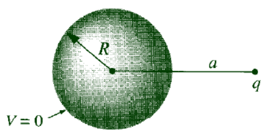

Figure 3.12

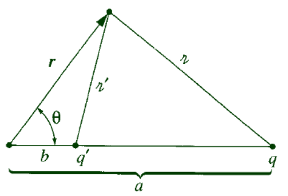

Figure 3.13

Solution: Examine the *completely different* configuration, consisting of the point charge q together with another point charge

$$q' = -\frac{R}{a}q, \tag{3.15}$$

Solution: Examine the *completely different* configuration, consisting of the point charge q together with another point charge

$$q' = -\frac{R}{a}q, \tag{3.15}$$

placed a distance

$$b = \frac{R^2}{a} \tag{3.16}$$

to the right of the center of the sphere (Fig. 3.13). No conductor, now—just the two point charges. The potential of this configuration is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{𝓇} + \frac{q'}{𝓇'}\right), \tag{3.17}$$

where $𝓇$ and $𝓇'$ are the distances from q and q', respectively. Now, it happens (see Prob. 3.7) that this potential vanishes at all points on the sphere, and therefore fits the boundary conditions for our original problem, in the exterior region.

Conclusion: Eq. 3.17 is the potential of a point charge near a grounded conducting sphere. (Notice that b is less than R, so the "image" charge q' is safely inside the sphere—*you cannot put image charges in the region where you are calculating* V; that would change ρ, and you'd be solving Poisson's equation with the wrong source.) In particular, the force of attraction between the charge and the sphere is

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq'}{(a-b)^2} = -\frac{1}{4\pi\epsilon_0}\frac{q^2Ra}{(a^2-R^2)^2}. \tag{3.18}$$

This solution is delightfully simple, but extraordinarily lucky. There's as much art as science in the method of images, for you must somehow think up the right "auxiliary problem" to look at. The first person who solved the problem this way cannot have known in advance