$$q(t) = \left[q_0 - \frac{\lambda}{m(\omega_d^2-\omega^2)}\right] cos(\omega t) + \frac{p_0}{m\omega} sin(\omega t) + \frac{\lambda cos(\omega_d t)}{m(\omega_d^2-\omega^2)} \quad (1)$$

$$p(t) = -m\omega \left[q_0 - \frac{\lambda}{m(\omega_d^2-\omega^2)}\right] sin(\omega t) + p_0 cos(\omega t) - \frac{\lambda \omega_d sin(\omega_d t)}{(\omega_d^2-\omega^2)} \quad (2)$$

with $(q_0 = q(0), p_0 = p(0))$ the initial data. Feeding this into the original Hamiltonian expression gives

$$\begin{aligned} E(q(t), p(t), t) \quad = \quad & \left\{ \left[\frac{1}{2m} p_0^2 + \frac{1}{2} m\omega^2 q_0^2\right] - \lambda\omega^2 q_0 \frac{1}{(\omega_d^2-\omega^2)} + \frac{\lambda^2(\omega^2+\omega_d^2)}{2m(\omega_d^2-\omega^2)^2} \right\} \quad (3) \\ & + \frac{\lambda^2}{2m(\omega_d^2-\omega^2)} \cos^2 t\omega_d + 2\omega p_0 \left[q_0 - \frac{\lambda}{m(\omega_d^2-\omega^2)}\right] \cos t\omega \sin t\omega \\ & + \frac{\lambda\omega_d^2[mq_0(\omega_d^2-\omega^2)-\lambda]}{m(\omega_d^2-\omega^2)^2} \cos t\omega \cos t\omega_d \\ & + \frac{\lambda\omega\omega_d[\lambda - mq_0(\omega_d^2-\omega^2)]}{m(\omega_d^2-\omega^2)^2} \sin t\omega \sin t\omega_d \\ & - \frac{\lambda\omega_d p_0}{m(\omega_d^2-\omega^2)} \cos t\omega \sin t\omega_d + \frac{\lambda p_0 \omega_d^2}{m\omega(\omega_d^2-\omega^2)} \cos t\omega_d \sin t\omega; \end{aligned}$$