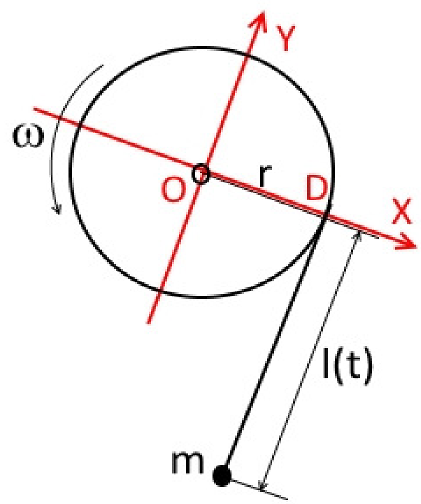

Рис. 1. ...

A disk of radius r rotates with constant angular velocity $\omega > 0$ about its axis. The axis of the disk is fixed. A stretchless and massless rope is wound around the disk. At the end of the rope a particle of mass m is attached. All the construction is on the horizontal smooth table. The length of straight part of the rope is $l = l(t)$. The rope is wound in opposite direction to the rotation of the disk. Study the motion of the system for which the rope remains strained.

Introduce a rotating coordinate frame $OXYZ$ with origin at the center of the disk and such that the axis OX passes through the point D where the rope is detouched off the disk. The axis OZ is perpendicular to the table and directed upwards. Then the angular velocity of the disk is $\boldsymbol{\omega} = \omega \boldsymbol{e}_z$. Let $\boldsymbol{\Omega} = \Omega \boldsymbol{e}_z$ stand for an angular velocity of the system $OXYZ$.

Relative angular velocity of the disk is $\boldsymbol{\omega}_r = \boldsymbol{\omega} - \boldsymbol{\Omega}$.

The radius-vector of the particle is $\boldsymbol{Om} = -l(t)\boldsymbol{e}_y + r\boldsymbol{e}_x$. The velocity of the particle is

$$\boldsymbol{v} = \boldsymbol{v}_e + \boldsymbol{v}_r,$$

where

$$\boldsymbol{v}_r = -\dot{l}\boldsymbol{e}_y = \boldsymbol{\omega}_r \times \boldsymbol{OD} = r(\omega - \Omega)\boldsymbol{e}_y, \quad \boldsymbol{OD} = r\boldsymbol{e}_x$$

is the relative velocity and

$$\boldsymbol{v}_e = \boldsymbol{\Omega} \times \boldsymbol{Om} = l\Omega \boldsymbol{e}_x + r\Omega \boldsymbol{e}_y$$

is the transport velocity. Particularly, one has $-\dot{l} = r(\omega - \Omega)$.

So that the kinetic energy is

$$T = \frac{m}{2}|\boldsymbol{v}|^2 = \frac{m}{2}\Big((r\omega)^2 + l^2(\omega + \dot{l}/r)^2\Big).$$

After getting rid of the time total derivative we obtain the following Lagrangian:

$$L(l, \dot{l}) = \frac{m}{2}\Big(l^2\omega^2 + \dot{l}^2 l^2/r^2\Big).$$