

On p. 180 of *La leçon de Platon*, by Dom Néroman (La Bégude de Mazenc, Arma Artis, 2002), the author presents these two equations:

$$x^2 + 2xy = M,$$

$$x^2 + y^2 = N,$$

and, on the next page, he says that those are the equations of a circle and of a hyperbola with the same center at the origin of the axes of coordinates. He adds that it is easy to find

$$x^2 = \frac{M + 2N \pm 2\sqrt{N^2 + MN - M^2}}{5},$$

$$y^2 = \frac{2M - N \pm \sqrt{N^2 + MN - M^2}}{5x}.$$

To begin with, I suspect that there is a misprint in the denominator of the last equation, which should be 5, instead of 5x; but I cannot be sure yet. Anyway, my problem is that I cannot (that is, I do not know how to) find

$$x^2 = \frac{M + 2N \pm 2\sqrt{N^2 + MN - M^2}}{5},$$

$$y^2 = \frac{2M - N \pm \sqrt{N^2 + MN - M^2}}{5x}$$

from the system

$$x^2 + 2xy = M,$$

$$x^2 + y^2 = N;$$

moreover, I have tried substituting:

a) $\frac{M+2N \pm 2\sqrt{N^2+MN-M^2}}{5}$ for x^2 , and $\frac{2M-N \pm \sqrt{N^2+MN-M^2}}{5}$ for y^2 in $x^2 + y^2 = N$;

b) $\frac{M+2N \pm 2\sqrt{N^2+MN-M^2}}{5}$ for x^2 , $\sqrt{\frac{M+2N \pm 2\sqrt{N^2+MN-M^2}}{5}}$ for x , and $\sqrt{\frac{2M-N \pm \sqrt{N^2+MN-M^2}}{5}}$ for y in

$$x^2 + 2xy = M,$$

and I cannot reduce the result to either N , in the first case, or to M , in the second one.