Differentiation theorem of x!

By:

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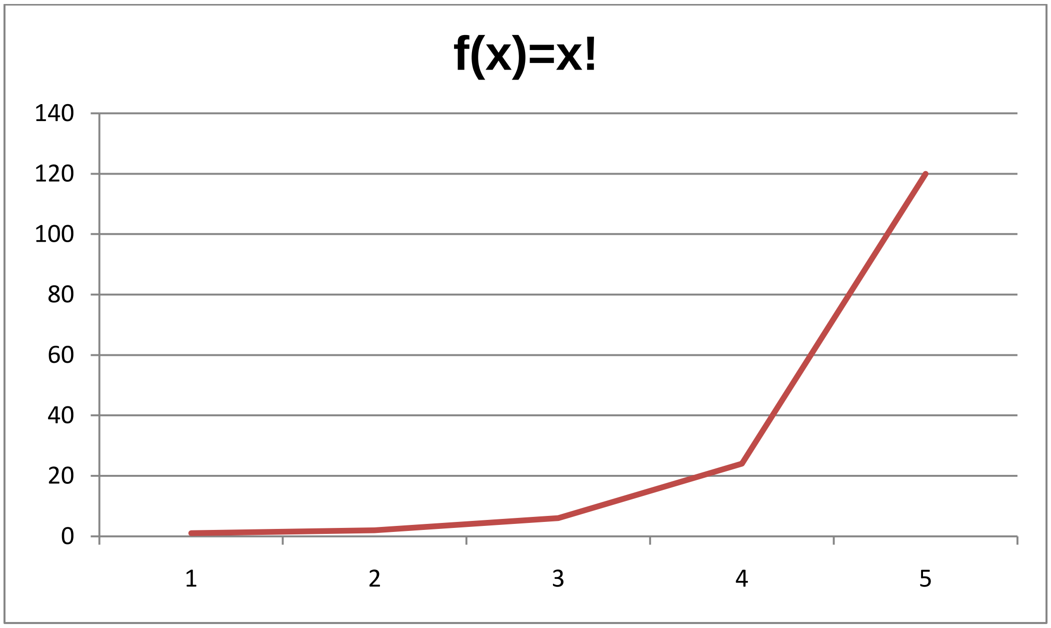
x	f(x)=x!
1	1
2	2
3	6
4	24
5	120



$$f(x) = x!$$

$$f'(x) = \lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$f'(x)= \lim_{\delta x \to 0} \frac{(x+\delta x)! - x!}{\delta x}$$

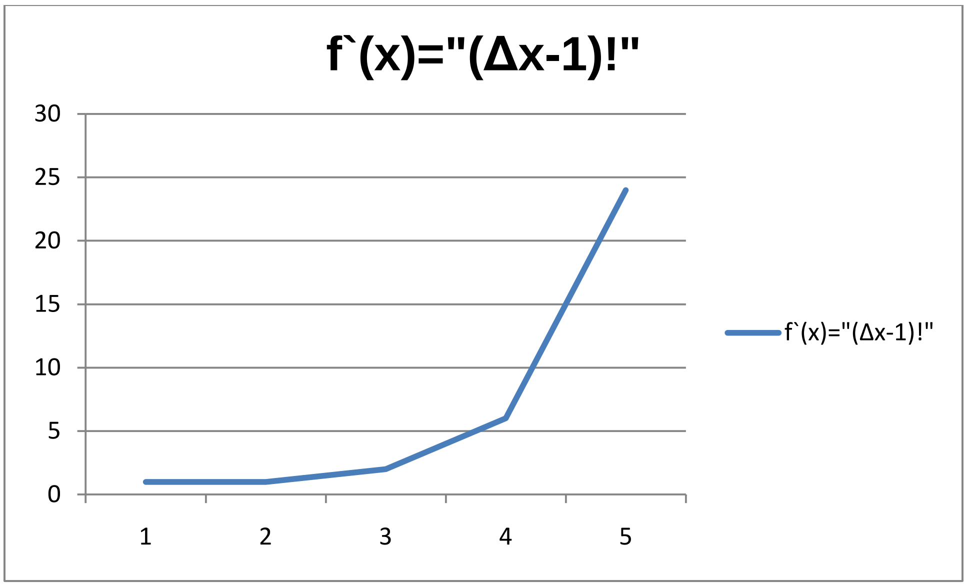
$$f'(x) = \lim_{\delta x \to 0} \frac{x!\delta x! - x!}{\delta x}$$

$$f'(x) = \lim_{\delta x \to 0} \frac{\delta x!}{\delta x}$$

$$f'(x) = \lim_{\delta x \to 0} (\delta x - 1)!$$

$$\frac{\delta y}{\delta x} = (x-1)!$$

Δx	(Δx-1)!
1	1
2	1
3	2
4	6
5	24



$$f''(x) = \lim_{\delta x \to 0} (x-1)!$$

$$f'''(x) = \lim_{\delta x \to 0} \frac{f''(\delta x + \delta\delta x) - f''(\delta x)}{\delta\delta x}$$

$$f'''(x) = \lim_{\delta x \to 0} \frac{(\delta x + \delta\delta x - 1)! - (\delta x - 1)!}{\delta\delta x}$$

$$f'''(x) = \lim_{\delta x \to 0} \frac{\delta\delta x!}{\delta x}$$

$$f'''(x) = \lim_{\delta x \to 0} (\delta\delta x - 1)!$$

$$\frac{\delta\delta y}{\delta\delta x} = (\delta\delta x - 1)!$$

$$f''''''\ldots''''''(x) = (\delta\delta\delta\ldots\delta\delta\delta\delta x - 1)!$$

Where there is an (n+1) number of “'”s and an “n” number of “δ”s.

$$\frac{\delta\delta\delta\ldots\delta\delta\delta y}{\delta\delta\delta\ldots\delta\delta\delta x} = (\delta\delta\delta\ldots\delta\delta\delta\delta x - 1)!$$

Where there are n “δ”s on both sides. Finally:

$$\frac{d^n y}{d^n x} = (d^n x - 1)!$$