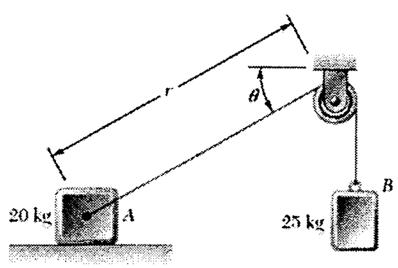
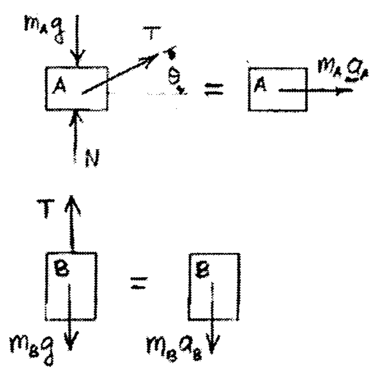
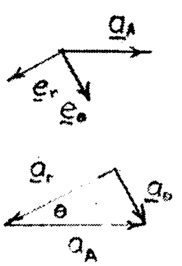


PROBLEM 12.72

The two blocks are released from rest when $r = 0.8$ m and $\theta = 30°$. Neglecting the mass of the pulley and the effect of friction in the pulley and between block A and the horizontal surface, determine (*a*) the initial tension in the cable, (*b*) the initial acceleration of block A, (*c*) the initial acceleration of block B.

SOLUTION





Let r and θ be polar coordinates of block A as shown, and let y_B be the position coordinate (positive downward, origin at the pulley) for the rectilinear motion of block B.

Constraint of cable: $r + y_B = \text{constant},$

$$\dot{r} + v_B = 0, \quad \ddot{r} + a_B = 0 \qquad \text{or} \qquad \ddot{r} = -a_B \quad (1)$$

For block A, $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T\cos\theta = m_A a_A$ or $T = m_A a_A \sec\theta$ (2)

For block B, $+\!\downarrow \Sigma F_y = m_B a_B$: $m_B g - T = m_B a_B$ (3)

Adding Eq. (1) to Eq. (2) to eliminate T, $m_B g = m_A a_A \sec\theta + m_B a_B$ (4)

Radial and transverse components of $\mathbf{a}_A$.

Use either the scalar product of vectors or the triangle construction shown, being careful to note the positive directions of the components.

$$\ddot{r} - r\dot{\theta}^2 = a_r = \mathbf{a}_A \cdot \mathbf{e}_r = -a_A \cos\theta \qquad (5)$$

Noting that initially $\dot{\theta} = 0$, using Eq. (1) to eliminate $\ddot{r}$, and changing signs gives

$$a_B = a_A \cos\theta \qquad (6)$$

Substituting Eq. (6) into Eq. (4) and solving for a_A,

$$a_A = \frac{m_B g}{m_A \sec\theta + m_B \cos\theta} = \frac{(25)(9.81)}{20\sec 30° + 25\cos 30°} = 5.48 \text{ m/s}^2$$

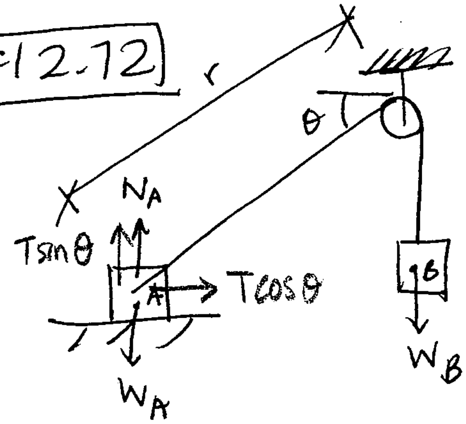
From Eq. (6), $a_B = 5.48\cos 30° = 4.75 \text{ m/s}^2$

(*a*) From Eq. (2), $T = (20)(5.48)\sec 30° = 126.6$ $\quad T = 126.6 \text{ N}$ ◀

(*b*) Acceleration of block A. $\quad \mathbf{a}_A = 5.48 \text{ m/s}^2 \longrightarrow$ ◀

(*c*) Acceleration of block B. $\quad \mathbf{a}_B = 4.75 \text{ m/s}^2 \downarrow$ ◀

#12.72



$W_A = 20\,kg(9.81) = 196.2\,N$

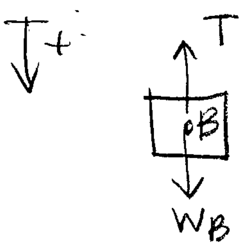
$W_B = 25\,kg(9.81) = 245.25\,N$

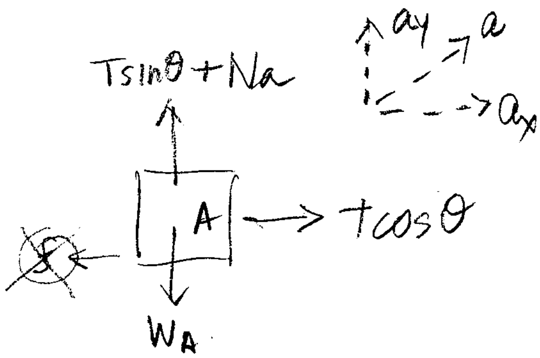
$V_{A,0} = V_{B,0} = 0$ & $r = 0.8m$, $\theta = 30°$

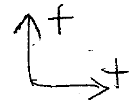
μ_f, pulley mass & $f \rightarrow$ neglect

Find: T_0, $a_{A,0}$; $a_{B,0}$

FBD's





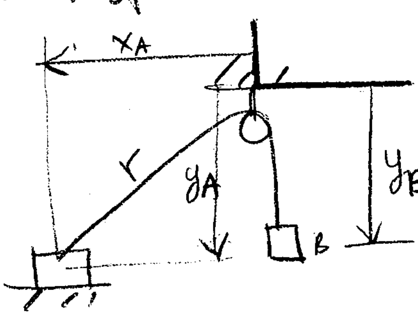


$-T + W_B = m_B a_B$

$\Sigma F_y = ma_y : T\sin\theta + Na - W_A = m_A a_{y,a}$

$\Sigma F_x = ma_x : T\cos\theta = m_A a_{x,a}$

Constraint of cable:



$r + y_B = constant$

$r = -y_B$

$\downarrow$

$\dot{r} = -V_B$

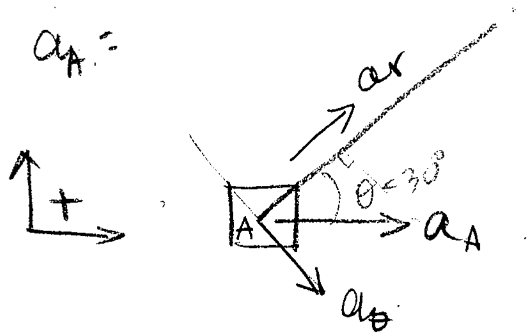
$\ddot{r} = -a_B$

Using radial & transverse components.

$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2)$

$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta})$

a_A:



$a_r = (\ddot{r} - r\dot{\theta}^2)$

$a_\theta = (r\ddot{\theta} + 2\dot{r}\dot{\theta})$

From equilibrium equations:

$-T + W_B = m_B a_{B,y}$ ——— (1)

$T\sin\theta + N_A - W_A = m_A a_{y,a}$ — (2)

$T\cos\theta = m_A a_{A,x}$ ——— (3).

Using (1) + (3)

(1) → $W_B - m_B a_B = T$. plug into (3)

$(W_B - m_B a_B)\cos\theta = m_A a_{A,x}$ ——— (4)

→ $a_r = a_A \cos\theta$

$a_A = \dfrac{(\ddot{r} - r\dot{\theta}^2)}{\cos\theta}$

we know: $a_B = -\ddot{r}$

$a_A = \dfrac{(-a_B - r\dot{\theta}^2)}{\cos\theta}$

$$a_A = \frac{(-a_B - \cancel{r\dot{\theta}^2})}{\cos\theta}$$

since $\theta = 30°$ initially and we are solving for the initial condition.

$$a_A = \frac{-a_B}{\cos\theta}$$

$$\frac{d(30)}{dt} = 0 = \dot{\theta}$$

plug a_B into (4), $a_B = -a_A\cos\theta$

$$(W_B - m_B(-a_A\cos\theta))\cos\theta = m_A a_A$$

solve for a_A: $a_A = 169.91\ m/s^2$

Why is my sign worong?

For T:

$$T = W_B - m_B a_B$$

$$T = W_B - m_B(-a_A\cos\theta)$$

θ	$T = W_B - m_B(-a_A\cos\theta)$
0	
15	
30	
45	
60	
90	