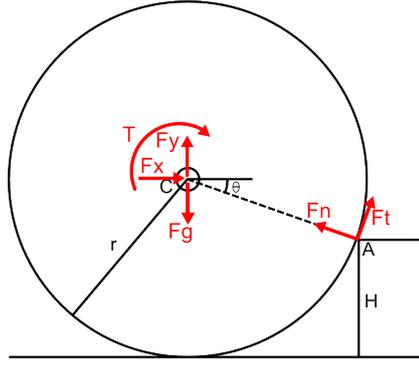


First the Free-Body-Diagram for the wheel. Since the wheel would presumably be lifted, there's no contact point with the ground. There is a reaction force in point A, with a normal and a tangent component. The normal reaction force doesn't contribute to the moment around the center point,  $M_C$ , as it's directed towards the center. There is also a resulting reaction force in point C, with an x and y component. Since the wheel will be rotating around point A, I assume this force must be directed normal to line A-C. Hence,  $F_x$  and  $F_y$  must be related by equation (4).



**Equations of Motion:**

$$M_C = T - F_t r \quad (1)$$

$$F_x = F_t \sin(\theta) - F_n \cos(\theta) \quad (2)$$

$$F_y = F_t \cos(\theta) + F_n \sin(\theta) - F_g \quad (3)$$

$$\tan(\theta) = \frac{F_x}{F_y} \quad (4)$$

$$M_A = T + F_y r \cos(\theta) + F_x r \sin(\theta) \quad (5)$$

Figure 1: Free Body Diagram Wheel

**Solving:** There are now 5 equations and 6 unknown variables ( $M_C$ ,  $F_x$ ,  $F_y$ ,  $F_n$ ,  $F_t$  and  $M_A$ ). So we set  $M_C$  as 0, which then allows us to solve the equations. From equation (1), (3) and (4):

$$F_t = T/r \quad (6)$$

$$F_n = \frac{F_y + F_g - F_t \cos(\theta)}{\sin(\theta)} \Rightarrow \frac{\frac{F_x}{\tan(\theta)} + F_g - F_t \cos(\theta)}{\sin(\theta)} \quad (7)$$

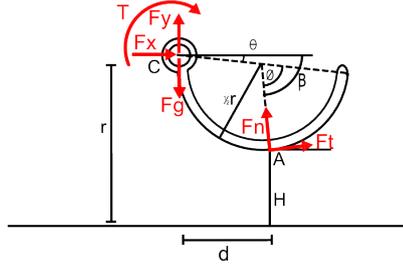
Inserting (7) into (2):

$$\begin{aligned} F_x &= F_t \sin(\theta) - \frac{\cos(\theta)}{\sin(\theta)} \left( \frac{F_x}{\tan(\theta)} + F_g - F_t \cos(\theta) \right) \\ &\Rightarrow F_t \sin(\theta) - \frac{\cos(\theta)}{\sin(\theta)} \frac{F_x}{\tan(\theta)} - \frac{\cos(\theta)}{\sin(\theta)} F_g + \frac{\cos(\theta)}{\sin(\theta)} F_t \cos(\theta) \\ &\Rightarrow \left( F_t \left( \sin(\theta) + \frac{\cos(\theta)}{\sin(\theta)} \cos(\theta) \right) - \frac{\cos(\theta)}{\sin(\theta)} F_g \right) / \left( 1 + \frac{\cos(\theta)}{\sin(\theta) \tan(\theta)} \right) \end{aligned} \quad (8)$$

$\theta$  is calculated from  $\sin(\theta) = \frac{r-H}{r}$  and with  $F_x$ , I can calculate  $F_y$  and hence  $M_A$ . With Excel I can then plot  $M_A$  against H, so no need to deduct an explicit formula.

For the C-leg, I'm mostly using the same procedure, although the deductions are more elaborate. There's also another variable  $d$  for the horizontal distance between C and A. For the wheel,  $d$  was determined by  $H$  and the rim of the wheel, but for the leg  $d$  can be freely chosen.

### Equations of Motion:



$$M_C = T + F_n \cos(\beta)(r - H) - F_n \sin(\beta)d - F_t \sin(\beta)(r - H) - F_t \cos(\beta)d \quad (9)$$

$$F_x = F_t \sin(\beta) - F_n \cos(\beta) \quad (10)$$

$$F_y = F_t \cos(\beta) + F_n \sin(\beta) - F_g \quad (11)$$

$$\tan(\alpha) = \frac{F_x}{F_y} = \frac{r - H}{d} \quad (12)$$

$$M_A = T + F_y d + F_x (r - H) \quad (13)$$

**Solving:** First simplifying equation (9):

$$\begin{aligned} M_C &= T + F_n (\cos(\beta)(r - H) - \sin(\beta)d) - F_t (\sin(\beta)(r - H) + \cos(\beta)d) \\ &= T + F_n C_1 - F_t C_2 \end{aligned}$$

With  $C_1 = \cos(\beta)(r - H) - \sin(\beta)d$  and  $C_2 = \sin(\beta)(r - H) + \cos(\beta)d$ .

Enter eq.(12) into eq.(11), combine with eq.(10), then solve to obtain  $F_n$ :

$$\begin{aligned} F_t \sin(\beta) - F_n \cos(\beta) &= (F_t \cos(\beta) + F_n \sin(\beta) - F_g) \tan(\beta) \\ \Rightarrow F_n \left( \frac{-\cos(\beta)}{\tan(\alpha)} - \sin(\beta) \right) &= F_t \left( \frac{-\sin(\beta)}{\tan(\alpha)} + \cos(\beta) \right) - F_g \\ \Rightarrow F_n &= \frac{F_t C_3}{C_4} - \frac{F_g}{C_4} \end{aligned} \quad (14)$$

Where  $C_3 = \frac{-\sin(\beta)}{\tan(\alpha)} + \cos(\beta)$  and  $C_4 = \frac{-\cos(\beta)}{\tan(\alpha)} - \sin(\beta)$ .

Now enter eq.(14) into eq.(9) and again setting  $M_C$  to be 0, solving for  $F_t$ :

$$\begin{aligned} M_C &= T + \left( \frac{F_t C_3}{C_4} - \frac{F_g}{C_4} \right) C_1 - F_t C_2 \\ 0 &= T + F_t \left( \frac{C_3 C_1}{C_4} - C_2 \right) - \frac{C_1 F_g}{C_4} \\ \Rightarrow F_t &= \left( \frac{C_1 F_g}{C_4} - T \right) / \left( \frac{C_3 C_1}{C_4} - C_2 \right) \end{aligned} \quad (15)$$

With  $F_t$ ,  $F_n$ ,  $F_x$ ,  $F_y$  and thus  $M_A$  can be calculated.

Still need to determine  $\beta$ , which turned out to be more difficult than I imagined.  $\beta$  is the sum of  $\phi$  and  $\theta$ .  $\phi$  can be found through the length between C and A.

$$\begin{aligned}
|C - A| &= \sqrt{d^2 + (r - H)^2} = \sqrt{\left(\frac{1}{2}r + \cos(\phi)\frac{1}{2}r\right)^2 + \left(\sin(\phi)\frac{1}{2}r\right)^2} \\
&\Rightarrow d^2 + (r - H)^2 = \left(\frac{1}{2}r + \cos(\phi)\frac{1}{2}r\right)^2 + \left(\sin(\phi)\frac{1}{2}r\right)^2 \\
&\Rightarrow d^2 + (r - H)^2 = \frac{1}{4}r^2 (\sin^2(\phi) + 1 + 2\cos(\phi) + \cos^2(\phi)) \\
&\Rightarrow d^2 + (r - H)^2 = \frac{1}{2}r^2 + \frac{1}{2}r^2\cos(\phi) \\
&\Rightarrow \cos(\phi) = \left(d^2 + (r - H)^2 - \frac{1}{2}r^2\right) / \frac{1}{2}r^2 \tag{16}
\end{aligned}$$

And  $\theta$  can be found with distance  $r$  and height  $r - H$ .

$$\begin{aligned}
d &= \frac{1}{2}r\cos(\theta) + \frac{1}{2}r\cos(\beta) \\
&\Rightarrow \frac{1}{2}r \left(2\cos\left(\frac{\theta + \beta}{2}\right)\cos\left(\frac{\theta - \beta}{2}\right)\right) \\
&\Rightarrow r\cos(A)\cos(B) \tag{17}
\end{aligned}$$

$$\begin{aligned}
r - H &= \frac{1}{2}r\sin(\theta) + \frac{1}{2}r\sin(\beta) \\
&\Rightarrow \frac{1}{2}r \left(2\sin\left(\frac{\theta + \beta}{2}\right)\cos\left(\frac{\theta - \beta}{2}\right)\right) \\
&\Rightarrow r\sin(A)\cos(B) \tag{18}
\end{aligned}$$

Where  $A = \frac{\theta + \beta}{2}$  and  $B = \frac{\theta - \beta}{2}$ . We can now combine eq.(17) and (18) and deduct  $\theta$ .

$$\begin{aligned}
\frac{d}{r\cos(A)} &= \frac{r - H}{r\sin(A)} \\
\Rightarrow dr\sin(A) &= (r - H)r\cos(A) \\
\Rightarrow \tan(A) &= \frac{r - H}{d} \\
\Rightarrow A = \frac{\theta + \theta + \phi}{2} &= \tan^{-1}\left(\frac{r - H}{d}\right) \\
\Rightarrow \theta &= \tan^{-1}\left(\frac{r - H}{d}\right) - \frac{1}{2}\phi \tag{19}
\end{aligned}$$

I'm now able to plot  $M_A$  versus height H for both bodies. For the C-leg, d was taken so the obstacle hits the middle point of the curve,  $d = \cos(\alpha)L$  with  $L = r\sqrt{\frac{1}{2}}$  and  $\sin(\alpha) = \frac{r-H}{L}$ . Other variables  $r=100\text{mm}$ ,  $T=500\text{Nmm}$  and  $F_g=20\text{N}$ . The plot shows that the C-leg has a higher  $M_A$ , which is what I want. A positive  $M_A$  means the body is able to overcome the object, so the wheel can climb a maximum of 15mm, whereas the C-leg can climb a maximum of 50mm.

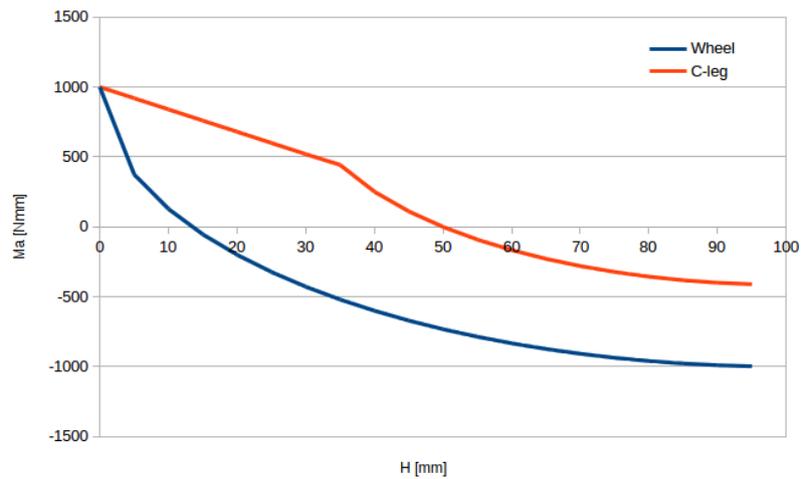


Figure 3: Plotting  $M_A$  vs H