

Figure 7.13

(see inset in Fig. 7.13). Therefore

$$\frac{d\Phi}{dt} = \oint \mathbf{B} \cdot (\mathbf{v} \times d\mathbf{l}).$$

Since $\mathbf{w} = (\mathbf{v} + \mathbf{u})$ and \mathbf{u} is parallel to $d\mathbf{l}$, we can also write this as

$$\frac{d\Phi}{dt} = \oint \mathbf{B} \cdot (\mathbf{w} \times d\mathbf{l}).$$

Now, the scalar triple-product can be rewritten:

$$\mathbf{B} \cdot (\mathbf{w} \times d\mathbf{l}) = -(\mathbf{w} \times \mathbf{B}) \cdot d\mathbf{l},$$

so

$$\frac{d\Phi}{dt} = - \oint (\mathbf{w} \times \mathbf{B}) \cdot d\mathbf{l}.$$

But $(\mathbf{w} \times \mathbf{B})$ is the magnetic force per unit charge, \mathbf{f}_{mag} , so

$$\frac{d\Phi}{dt} = - \oint \mathbf{f}_{\text{mag}} \cdot d\mathbf{l},$$

and the integral of \mathbf{f}_{mag} is the emf

$$\mathcal{E} = - \frac{d\Phi}{dt}. \quad \text{qed}$$

There is a sign ambiguity in the definition of emf (Eq. 7.9): Which *way* around the loop are you supposed to integrate? There is a compensatory ambiguity in the definition of *flux* (Eq. 7.12): Which is the positive direction for $d\mathbf{a}$? In applying the flux rule, sign consistency is governed (as always) by your right hand: If your fingers define the positive direction around the loop, then your thumb indicates the direction of $d\mathbf{a}$. Should the emf come out negative, it means the current will flow in the negative direction around the circuit.

The flux rule is a nifty short-cut for calculating motional emf's. It does *not* contain any new physics. Occasionally you will run across problems that cannot be handled by the flux rule; for these one must go back to the Lorentz force law itself.

Example 7.4

A metal disk of radius a rotates with angular velocity ω about a vertical axis, through a uniform field \mathbf{B} , pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact, which touches the outer edge of the disk (Fig. 7.14). Find the current in the resistor.

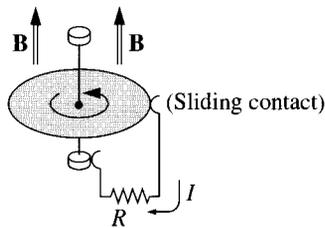


Figure 7.14

Solution: The speed of a point on the disk at a distance s from the axis is $v = \omega s$, so the force per unit charge is $\mathbf{f}_{\text{mag}} = \mathbf{v} \times \mathbf{B} = \omega s B \hat{s}$. The emf is therefore

$$\mathcal{E} = \int_0^a \mathbf{f}_{\text{mag}} ds = \omega B \int_0^a s ds = \frac{\omega B a^2}{2},$$

and the current is

$$I = \frac{\mathcal{E}}{R} = \frac{\omega B a^2}{2R}.$$

The trouble with the flux rule is that it assumes the current flows along a well-defined path, whereas in this example the current spreads out over the whole disk. It's not even clear what the "flux through the circuit" would *mean* in this context. Even more tricky is the case of **eddy currents**. Take a chunk of aluminum (say), and shake it around in a nonuniform magnetic field. Currents will be generated in the material, and you will feel a kind of "viscous drag"—as though you were pulling the block through molasses (this is the force I called \mathbf{f}_{pull} in the discussion of motional emf). Eddy currents are notoriously difficult to calculate,⁵ but easy and dramatic to demonstrate. You may have witnessed the classic experiment in which an

⁵See, for example, W. M. Saslow, *Am. J. Phys.*, **60**, 693 (1992).

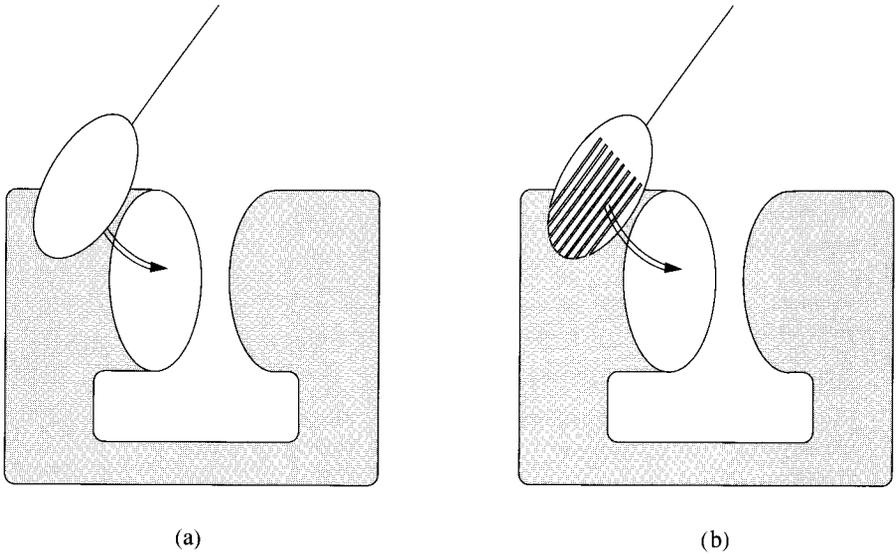


Figure 7.15

aluminum disk mounted as a pendulum on a horizontal axis swings down and passes between the poles of a magnet (Fig. 7.15a). When it enters the field region it suddenly slows way down. To confirm that eddy currents are responsible, one repeats the process using a disk that has many slots cut in it, to prevent the flow of large-scale currents (Fig. 7.15b). This time the disk swings freely, unimpeded by the field.

Problem 7.7 A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (Fig. 7.16). A resistor R is connected across the rails and a uniform magnetic field \mathbf{B} , pointing into the page, fills the entire region.

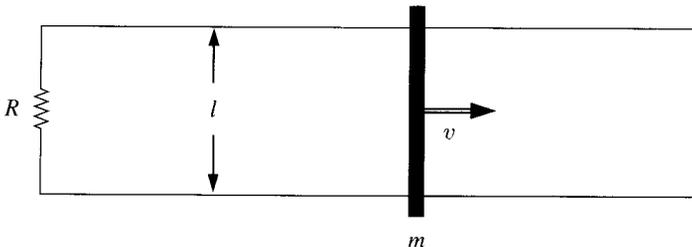


Figure 7.16