

PGE382L — CP4

Solve the following partial differential equation using *Crank-Nicolson* method. For the space discretization use three-point central difference approximations with $\Delta x = 1/40$ and $t_f = .5$.

$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} + .003 \frac{\partial^2 u}{\partial x^2}, \quad x \in (0,1), \quad t > 0$$

Assume initial and boundary conditions of the form

$$u(0,x) = \phi(0,x), \quad u(t,0) = \phi(t,0), \quad u(t,1) = \phi(t,1),$$

where

$$\phi(t,x) = (.1e^{-A} + .5e^{-B} + e^{-C}) / (e^{-A} + e^{-B} + e^{-C}),$$

and

$$A = \frac{50}{3}(x - .5 + 4.95t)$$

$$B = \frac{250}{3}(x - .5 + .75t)$$

$$C = \frac{500}{3}(x - .375)$$