Introduction

If we examine the content of a set in terms of the symmetry concept, we can find at least two levels of symmetries that can be ordered by their simplicity degrees.

The most symmetrical and simplest content is Emptiness, which is represented by the empty set notation { } = content does not exist.

On top of this simplicity we can define two opposite types of symmetry contents,
{__} and {._. .} .

Let power ^{0} be the simplest level of existence of some set's content.

{__} is an infinite non-localized element that notated as $0^0 = 1$ (1 continuum)

{._. .} is infinitely many elements that are notated as $\infty^0 = 1$ (connector XOR point)

So, what we get is this basic information structure: {._. .}   {___}
                                                                                { }

Let { } be ***E*** simplicity or ***E***sim (***E*** for Emptiness).

Let {___} be ***C***sim (***C*** for Continuum).

Let {._. .} be ***D***sim (***D*** for Discreteness).

~ = NOT

Any transformation from {} to {__} or {._. .} is based on phase transition, because we have |{}|(=0) to ~|{}|(=~0) transition.

A ***C***sim and a ***D***sim are opposites because ***C***sim is a one continuum and ***D***sim is finite or infinitely many elements.

The above identification is based on the structure property, and it can not be done by the quantity property, because ***C***sim XOR ***D***sim are exclusively = 1 .

So, in the case of the symmetry concept, the structure property is more sensitive than the quantity property, when we examine them by the information concept.

If we want to go beyond the information about the existence of ***C***sim XOR ***D***sim,
we have to associate between them, by changing XOR to AND connective.

By doing this we can define elements that have properties, which are combinations of ***E***sim, ***C***sim and ***D***sim.

Let us find a definition for existence under Complementary Associations Theory (CAT) :

Existence

Definition AA: Un-explorable Existence is a state of some opposite concepts,
before there is any mutual influence on each opposite's property.

Example: Light before turning into darkness, darkness before turning into light.

Definition BB: Explorable Existence is a state of some opposite concepts,
where there is a mutual influence between their opposite properties.

Example: Light turning into darkness, darkness turning into light.

Let us write the CAT's axiom and definitions.

The Axiom of exploration:
Explorable is any association between ***C***sim AND ***D***sim.

Definition A:
Explorable Product (***EP***) exists iff it is an association between Continuum (***C***sim) and Discreteness (***D***sim) concepts, so ***CD*** is ***C***sim AND ***D***sim .

Now, let us answer some questions:

Q: What is an "association"?
A: Association is any possible mutual influence between opposite concepts.

Q: What is an "explorable product"?
A: According to definition BB, it is the element coming from association,
and it can be explored.

Definition B:
Association-Level (***AL***) is an invariant quantity, being kept through ***CD*** associations.

Definition C:
Computational Root (***CR***) is ***EP*** in ***AL***.

(An example of definitions B and C:

```
.   .   .      .   .   .      .   .   .
|   |   |      |   |   |      |   |   |
|   |   |      |___|_  |      |___|   |
|   |   |      |       |      |       |
|___|___|_     |_______|      |_______|
|              |              |
```

CR quantity is being kept through ***CD*** associations)

<u>Definition D:</u>
Redundancy and Uncertainty (***RU***) concepts, are used as invariant structural degree of ***CR***, determining its exact position in ***AL*** (there is an algorithm for this).

<u>Definition E:</u>
Full ***RU*** (***FRU***) is the first ***CR*** in ***AL***.

<u>Definition F:</u>
Not ***RU*** (~***RU***) is the last ***CR*** in ***AL***.

<u>Definition G:</u>
Partial ***RU*** (***PRU***) is any ***CR*** which is not ***FRU*** and not ~***RU***.

An example of definitions E, F and G:

```
.   .   .      .   .   .      .   .   .
|   |   |      |   |   |      |   |   |
|   |   |      |___|_  |      |___|   |
|   |   |      |       |      |       |
|___|___|_     |_______|      |_______|
|              |              |
```

FRU CR ***PRU CR*** ~***RU CR***

<u>A general graphic description of a ***CR***</u>

```
.     .     .<------ D (Discreteness)
|     |     |
|     |     |
|     |     |<------ The association between CD
|     |     |
|     |     |
|_____|_____|__<---- RU marker
|  ^
|   \____ C (Continuum)
|
|<---- Next-AL marker
```

If we connect ideas coming from Information Theory and Topology, then we can use a concept like symmetry, to describe a connection between structure and information's clarity-degree, for example:

```
<-Redundancy->
 c   c   c  ^<----Uncertainty
 b   b   b  |    b   b
 a   a   a  |    a   a   c        a   b   c
 .   .   .  v    .   .   .        .   .   .
 |   |   |       |   |   |        |   |   |
 |   |   |       |___|_  |        |___|   |
 |   |   |       |       |        |       |
 |___|___|_      |_______|        |_______|
 |               |                |
 FRU CR          PRU CR           ~RU CR
```

The uncertainty is based on XOR connective between a,b,c,...

This example describes ***AL*** 3 and we can see the connection between structure's symmetry-degree and information's clarity-degree, determining ***CR***'s exact position in ***AL*** 3.

Because ***RU*** concepts are not used in ZF or Peano's axioms, all their number system is limited to ~***RU CR***, therefore their number system is a proper subset of CAT's number system.

Some claim that using sequences instead of sets can do it, but we use the ***RU*** concepts as the fundamentals of all information structures, where ~***RU*** is a private case of some information structure, that used as the information structure of ZF or Peano's axioms.

More than that, because we clearly distinguish between the Continuum and the Discreteness concepts, we can represent a number by using the familiar notations but with a new meaning, for example let us represent the number four:

```
    1  |        1  |          1  |            1  |              1
    +  |    2 = +  |      2 = +  |            +  |          2 = +
    1  |        1  |          1  |        3 = 1  |      3 = +   1
4 = +  |4 = +      |4 = +        |4 = +       +  |4 = +
    1  |        1  |          1  |            1  |              1
    +  |        +  |      2 = +  |               |
    1  |        1  |          1  |            1  |              1
```

In the above example we can see fades between the continuous side of number four, which is represented by a single notation (4), and the discrete side of number four, which is represented by four notations (1 1 1 1), where the quantity in each information structure of number four is being kept, and we can learn that the Structure concept is more sensitive than the Quantity concept in Math language.

CAT's number system representation

Now let us examine the number system representation of ***AL***s 0 to 4:

```
0 =  __   .  = {} (Before Association)
               *

      0
      .
1 =   |   (First Association between Continuum and Discreteness)
      *


    1   1
    0   0     0   1
    .   .     .   .
    |   |     |   |
2 = |___|_    |___|
    |         | *


    2   2   2
    1   1   1       1   1
    0   0   0       0   0   2       0   1   2
    .   .   .       .   .   .       .   .   .
    |   |   |       |   |   |       |   |   |
3 = |   |   |       |___|_  |       |___|   |
    |   |   |       |       |       |       |
    |___|___|_      |_______|       |_______|
    |               |               |   *


    -------------->>>
    3  3  3  3           3  3           3  3
    2  2  2  2           2  2           2  2
    1  1  1  1     1  1  1  1           1  1     1  1  1  1           1  1
    0  0  0  0     0  0  0  0     0  1  0  0     0  0  0  0     0  1  0  0
    .  .  .  .     .  .  .  .     .  .  .  .     .  .  .  .     .  .  .  .
    |  |  |  |     |  |  |  |     |  |  |  |     |  |  |  |     |  |  |  |
    |  |  |  |     |__|_ |  |     |__|  |  |     |__|_ |__|_    |__|  |__|_
    |  |  |  |     |     |  |     |     |  |     |     |        |     |
    |  |  |  |     |     |  |     |     |  |     |     |        |     |
    |__|__|__|_    |_____|__|_    |_____|__|_    |_____|____    |_____|____
    |              |              |              |              |

4 =                2  2  2
                   1  1  1        1  1
    0  1  0  1     0  0  0  3     0  0  2  3     0  1  2  3
    .  .  .  .     .  .  .  .     .  .  .  .     .  .  .  .
    |  |  |  |     |  |  |  |     |  |  |  |     |  |  |  |
    |__|  |__|     |  |  |  |     |__|_ |  |     |__|  |  |
    |     |        |  |  |  |     |     |  |     |     |  |
    |     |        |__|__|_ |     |_____|  |     |_____|  |
    |     |        |        |     |        |     |        |
    |_____|____    |________|     |________|     |________|
    |              |              |              |   *
```

The ***CR***s marked by `*` are number system representations, based on Peano's axioms.

Let us examine the structure of `*` representations, through CAT's eyes:

0 = { } = __ . (Before *CD* associations)

```
1 = {{ }} = {0}
      .
      |
```

```
2 = {{ },{{ }}} = {0,1}
      .    .
      |    |
      |____|
      |
```

```
3 = {{ },{{ }},{{ },{{ }}}} = {0,1,2}
      .    .     .    .
      |    |     |    |
      |____|     |____|
      |          |
      |__________|
      |
```

```
4 = {{ },{{ }},{{ },{{ }}},{{ },{{ }},{{ },{{ }}}}} = {0,1,2,3}
      .    .     .    .      .    .     .    .
      |    |     |    |      |    |     |    |
      |____|     |____|      |____|     |____|
      |          |           |          |
      |__________|           |__________|
      |                      |
      |______________________|
      |
      |
```

This Fractalic-Information-Structure (*FIS*) is based on

```
      .    .
      |    |
      |____|
      |
```

which is ~*RU CR* in *AL* 2 .

If we build the *FIS* by using FRU CR in AL 2, we get *FIS*

```
      .    .     .    .      .    .     .    .
      |    |     |    |      |    |     |    |
      |____|_    |____|_     |____|_    |____|_
      |          |           |          |
      |__________|______     |__________|______
      |                      |
      |______________________|_________________
      |
      |
```

which has a different structure of number system representation.

Let us examine how a number system representation, based on Peano's axioms, uses ~***RU CR*** to represent a number system which is based on base value expansion representation, and for example we shall use number 53 represented by base 10 and base 3:

Number 53 represented by base 10:

```
               ^0- 0123456789
                   ||||||||||
                   |_||||||||
                   |__|||||||
                   |___||||||
~RU CR in AL 10 =  |____|||||
                   |_____||||
                   |______|||
                   |_______||
                   |________|
               ^1- |
                   |                              1      0
                                           ( 5*10 + 3*10 )
^0- 012345678901234567890123456789012345678901234567890123
    ||||||||||||||||||||||||||||||||||||||||||||||||||||||
    |_|||||||||_|||||||||_|||||||||_|||||||||_|||||||||_||
    |__||||||||__||||||||__||||||||__||||||||__||||||||__|
    |___|||||||___|||||||___|||||||___|||||||___||||||__...
    |____||||||____||||||____||||||____||||||____|||||__...
    |_____|||||_____|||||_____|||||_____|||||_____||||__...
    |______||||______||||______||||______||||______|||__...
    |_______|||_______|||_______|||_______|||_______||__...
    |________||________||________||________||________|__...
^1- |    0    |    1    |    2    |    3    |    4    | 5
    |_________|         |         |         |         |
    |___________________|         |         |         |
    |_____________________________|         |         |
    |_______________________________________|         |
    |_________________________________________________|
    |_ ...
    |
```

Number 53 represented by base 3:

```
               ^0- 012
~RU CR in AL 3 =   |||
                   |_|
               ^1- |                   3     2     1     0
                                 ( 1*3 + 2*3 + 2*3 + 2*3 )
^0- 012012012012012012012012012012012012012012012012012012
    ||||||||||||||||||||||||||||||||||||||||||||||||||||||
    |_||_||_||_||_||_||_||_||_||_||_||_||_||_||_||_||_||_|
^1- |0 |1 |2 |0 |1 |2 |0 |1 |2 |0 |1 |2 |0 |1 |2 |0 |1 |2
    |__|  |  |__|  |  |__|  |  |__|  |  |__|  |  |__|  |
    |_____|  |_____|  |_____|  |_____|  |_____|  |_____|
^2- |  0     |  1     |  2     |  0     |  1     |  2
    |________|        |        |________|        |
    |_________________|        |_________________|
^3- |        0                 |        1
    |__________________________|
    |_ ...
    |
```

Now let us represent Continuum XOR Discreteness before they are explorable:

```
                                                           0
<--''''''''''''''''''''''''''''''''''''''''''''''''''''--> D = ∞   = 1 = Exists
                                                        (un-explorable)
                      No Association
                                                          0
<--____________________________________________________--> C = 0   = 1 = Exists
                                                        (un-explorable)
```

And after association we get the elementary information structure, which is the base of any representation of number 53:

```
                                              0         0
FRU CR in AL 54 = Association between  54*0  and  54*1
                                                                  0
''''''''''''''''''''''''''''''''''''''''''''''''''''''-> D = 54*1  = difference
||||||||||||||||||||||||||||||||||||||||||||||||||||||
||||||||||||||||||||||||||||||||||||||||||||||||||||||<----------- Association
||||||||||||||||||||||||||||||||||||||||||||||||||||||            0
______________________________________________________-> C = 54*0  = sum
```

By writing "Computational Root" we mean that a ***CR*** is the building block of any ***FIS*** (Fractalic-Information-Structure) that can be systematically explored.

Here are some ***FIS***es which are not ~***RU CR***:

```
c   c   c    c   c   c    c   c   c
b   b   b    b   b   b    b   b   b        b   b        b   b        b   b
a   a   a    a   a   a    a   a   a        a   a   c    a   a   c    a   a   c
.   .   .    .   .   .    .   .   .        .   .   .    .   .   .    .   .   .
|   |   |    |   |   |    |   |   |        |   |   |    |   |   |    |   |   |
|   |   |    |   |   |    |   |   |        |___|_  |    |___|_  |    |___|_  |
|   |   |    |   |   |    |   |   |        |       |    |       |    |       |
|___|___|_   |___|___|_   |___|___|_       |_______|    |_______|    |_______|
|   c        |   c        |   c            |   b        |   b        |   c
|   b        |   b        |   b            |   a        |   a        |
|   a        |   a        |   a            |____________|_______     |
|            |            |                |                         |
|            |            |                |                         |
|____________|____________|_________       |_________________________|
|                  c                       |            b
|                  b                       |            a
                   a
```

by using ***RU*** concepts when defining a number system, we are enriching the ways of using a number system representation.

CAT has a ***C***-bound 0^0 and a ***D***-bound ∞^0, giving us a mathematical universe which has infinite levels of structure's symmetry-degrees connected to infinite levels of information's clarity-degrees, over infinite scales (where some representations of it can be found in the next pages).

A detailed representation of *AL*s 1 to 5

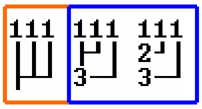
(some of the ***RU*** markers (please see page 2) have been changed by numerals)

```
1
--
(1)      -> 1
```

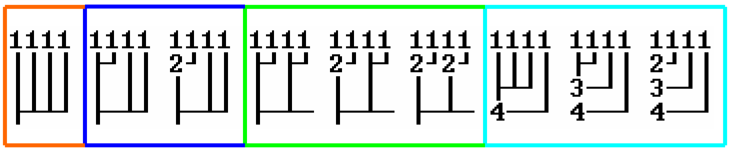
```
2
--
(1;1)*2 -> 2
```

```
11 11
   2┘
```

```
3
--
(1;1;1)*1  -> 1
 V │ V        +
(2«┘;1)*2  -> 2
              --
               3
```



```
4
--
(1;1;1;1)*1  -> 1
 V │ V V        +
(2«┘;1;1)*2  -> 2
 V   V │        +
(2   ;2«┘)*3 -> 3
 V              +
(3«┘┌┐└»;1)*3 -> 3
                --
                 9
```



```
5
--
(1;1;1;1;1)*1  -> 1
 V │ V V V        +
(2«┘;1;1;1)*2  -> 2
 V   V │ V        +
(2   ;2«┘;1)*3 -> 3
 V ┌┐    V        +
(3«┘└»;1;1)*3  -> 3
 V     V │        +
(3     ;2«┘)*6 -> 6
 V      ┌┐        +
(4«─────┘└»;1)*9 -> 9
                  --
                  24
```