Suppose we have a complete list of the real numbers between 0 and 1 in no particular order. To avoid confusion, do not allow a representation that ends in infinite 9s. Here it is shown in three columns: row number, real number, and real number expanded with space to make digits easier to examine.

1	0.727769558391632...	0 . 7 2 7 7 6 9 5 5 8 3 9 1 6 3 2 . . .
2	0.502520226786972...	0 . 5 0 2 5 2 0 2 2 6 7 8 6 9 7 2 . . .
3	0.802559796060574...	0 . 8 0 2 5 5 9 7 9 6 0 6 0 5 7 4 . . .
4	0.314497871880072...	0 . 3 1 4 4 9 7 8 7 1 8 8 0 0 7 2 . . .
5	0.556979021418631...	0 . 5 5 6 9 7 9 0 2 1 4 1 8 6 3 1 . . .
6	0.855454042073674...	0 . 8 5 5 4 5 4 0 4 2 0 7 3 6 7 4 . . .
7	0.296971875146792...	0 . 2 9 6 9 7 1 8 7 5 1 4 6 7 9 2 . . .
8	0.877669808530979...	0 . 8 7 7 6 6 9 8 0 8 5 3 0 9 7 9 . . .
9	0.407313625785182...	0 . 4 0 7 3 1 3 6 2 5 7 8 5 1 8 2 . . .
10	0.258261695563078...	0 . 2 5 8 2 6 1 6 9 5 5 6 3 0 7 8 . . .
.		
.		
.		

Now one can generate a number, not on that list by piecing together one digit at a time which is rigged to not match one number at a time on the list. For instance, place a 0 if the list digit is nonzero and 1 otherwise. The colors indicate which digits are associated with the digits on the list.

Generated number: 0 . 0 1 0 0 0 0 0 1 0 0 . . .

Clearly, this can continue forever and will define a real number that is not on the list. So the list is not complete and the reals between 0 and 1 are not countable. In fact, it is clear that one can make a great many omitted numbers using similar rules and that they are many more than are on the countable list. There are 9 digit-options that do not match the first number's digit, 9*9 digit-options that do not match the first or second number, 9*9*9 digit-options that do not match the first three numbers, etc. So there are an enormous quantity of numbers that have been missed – many more than are on the list. In fact, they are uncountable.