Boundary conditions at current interface

August 27, 2014

The source of the current is the polarizability of the particles. Current density at the interface is given by: $\boldsymbol{j} = -i\omega n_s \boldsymbol{p} = -i\omega n_s \overleftrightarrow{\boldsymbol{\alpha}}.\boldsymbol{E}$ where $\boldsymbol{E}$ is the electric field at the location of the particles. The polarizabiltiy $\boldsymbol{\alpha}$ is a tensor and it could be anisotropic.

1 Perpendicular polarized light

Suppose the incident light is of perpendicular polarization - in this case the electric field components will be perpendicular to the incident plane. Hence the dipole moment of the particles will also be perpendicular to the incident plane. The magnetic field will be in the plane of incidence.

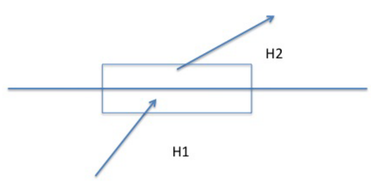

Figure 1: Application of boundary condition to perpendicularly polarized light

The boundary condition for the magnetic field is got from applying the surface integral of $\boldsymbol{H}$ over the rectangular loop shown in the figure and considering what happens when the thickness of the loop approaches zero. We get:

$$H_{t1} - H_{t2} = J_{-y}$$

and

$$E_{t1} = E_{t2}$$

Note the orientation of axes in this configuration:

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z     y(into the page, following the right hand rule)
|    /
|  /
|/ - - - - > x
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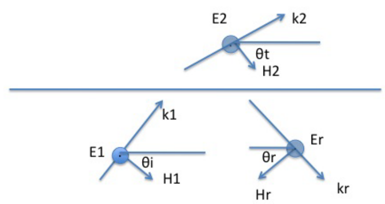

Figure 2: Application of boundary condition to perpendicularly polarized light

This gives:

$$\boxed{\begin{aligned} (H_i \cos\theta_i - H_r \cos\theta_i) - H_t \cos\theta_t = J_{-y} \\ E_i + E_r = E_t \end{aligned}}$$

where, J_{-y} denotes the component of the current in the $-y$ direction. Note that the positive y axis is into the plane of the paper as per the right hand rule. If we have air on both sides, we end up with $\cos\theta_i = \cos\theta_r = \cos\theta_t = k_z/k_0$ Using $E/H = \mu_0 c_0$ we get:

$$\frac{1}{\mu_0 c_0}\frac{k_z}{k_0}\big[\,(E_i - E_r) - E_t\big] = J_{-y}$$

$$E_i + E_r = E_t$$

Substituting the second equation in the first:

$$\frac{1}{\mu_0 c_0}\frac{k_z}{k_0}\big[\,(E_i - E_r) - (E_i + E_r)\big] = J_{-y}$$

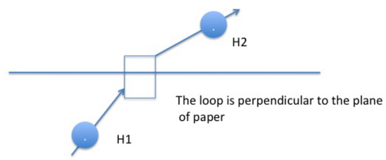

Figure 3: Application of boundary condition for magnetic field to parallel polarized light. The rectangular loop will be perpendicular to the page and we need to find the current passing through the loop

Dividing throughout by E_i and taking the incident wave to have unit amplitude at the interface, we get the reflection coefficient to be:

$$\hat{R} = -\frac{J_{-y}\mu_0\omega}{2k_z}$$

and the transmission coefficient as:

$$\hat{T} = 1 + \hat{R}$$

These two satisfy the energy conservation statement very well :

$$S_{1z} - S_{2z} = -\boldsymbol{J}.\boldsymbol{E}$$

where $\vec{S}_1$ and $\vec{S}_2$ are the Poynting vectors on either side of the interface and $\boldsymbol{E}$ is the total electric field (incident + reflected).

1.1 p-polarized (parallel polarized, TM polarized)

For parallel polarized light the current will be in the x-z plane but different from the orientation of the electric field (which will also be in the x-z plane) due to the effect of the polarizability tensor. Consider the case when the polarizability is such that the current is only along z-axis, even though the electric field has both x and z components.

The boundary conditions will then be (note that the magnetic field would depend only on component of $\boldsymbol{J}$ along x-axis. However as I mentioned above I am analyzing for the case when the current has only a z-component):

$$\boxed{\begin{aligned} (-E_i\cos\theta_i + E_r\cos\theta_r) &= -E_t\cos\theta_t \\ H_i + H_r - H_t &= 0 \end{aligned}}$$

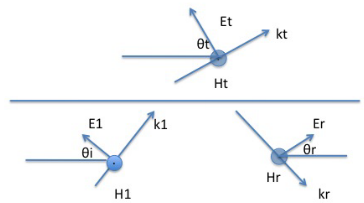

Figure 4: Application of boundary condition for parallel polarized light.

Using $E/H = \mu_0 c_0$ taking $\cos\theta_i = \cos\theta_r = \cos\theta_r = \dfrac{k_z}{k_0}$ we get:

$$
\begin{aligned}
(E_i \ - E_r) &= E_t \\
E_i + E_r &= E_t
\end{aligned}
$$

which gives us $1 - \hat{R} = 1 + \hat{R}$

Instead of taking the boundary condition of tangential electric field being continuous, if I take the Gaussian boundary condition:

$$E_{tz} - (E_{iz} + E_{rz}) = \frac{\sigma}{\epsilon_0}$$

with σ being the surface charge density. I get the two equations to be (with k_ρ being the inplane wave vector):

$$
\begin{aligned}
\frac{k_\rho}{k_0}\big[(E_i \ + E_r) - E_t\big] &= \frac{\sigma}{\epsilon_0} \\
E_i + E_r - E_t &= 0
\end{aligned}
$$

This is not solvabe too!

How do we obtain the reflection and transmission coefficient for such a case ?¿

And more importantly why is this method not working ? Does it have anything to do with the anisotropicity that is assumed ?