Derivation of the Born rule from outcome counting and a solution to the quantitative problem of the MWI

David Strayhorn

Washington University in Saint Louis
Saint Louis, MO, USA
wdstrayh@alumni.princeton.edu

RÉSUMÉ. (French translation of abstract goes here)

ABSTRACT. The "quantitative problem" of the MWI is to justify the interpretation of the Born rule measure $|a_n|^2$ *– the squared norm of the amplitude associated with the* n^{th} *out of* N *possible results – as a probability. The essential difficulty is that the basic framework of the MWI would seem to suggest an alternative probability rule, outcome counting, which is that each separate outcome should be equally likely. In this paper, a model is proposed that replaces the Born rule with outcome counting as the fundamental probability rule at the fine-grained level, and yet recovers the Born rule as a coarse-grained approximation. This model is proposed, not only as a solution to the quantitative problem, but also as a novel derivation of the Born rule.*

P.A.C.S.:

1 Introduction

In the Everett relative state formulation [1], which has come to be known as the multiple worlds interpretation (MWI), probability makes its appearance at the moment one assumes that probabilities are governed by the Born rule. According to the Born rule, a probability measure $m_n = |a_n|^2$ is assigned to each of the N experimental results associated with a measurement, with a_n being the complex coefficient of the n^{th} result as calculated by the Schrödinger equation. Despite the great deal of attention paid to the MWI since its introduction almost 50 years ago, several interpretational issues remain unresolved.

One of the more thorny interpretational issues is what has been called the "quantitative problem." The essential difficulty has been formulated and discussed by various authors. According to Graham, "it is extremely difficult to see what significance such a measure [the Born rule measure] can have when its implications are completely contradicted by a simple count of the worlds involved, worlds that Everett's own work assures us must all be on the same footing" [2]. More recently, Wallace has asked why the probability of a given branch should be quantified "according to the probability rule (the Born rule), and not (for instance) some other assignment of probabilities to branches?" [3] And Greaves notes that "The quantitative problem is perhaps unproblematic in the case of equally weighted superpositions, but has been considered fatal in the case of unequally weighted superpositions ... it seems that all Everett can say is that each outcome occurs in exactly one branch, which, we might think, *will yield equal probabilities if any at all*" [4] (emphasis mine).

A detailed discussion of the philosophical issues surrounding the quantitative problem – including whether it even *is* a problem – is beyond the scope of this paper[1]. Instead, this paper adopts the viewpoint expressed by Greaves above, and starts from the assumption that the best and only solution to the quantitative problem is simply to find a model that assumes "equally weighted superpositions" in which "each outcome ... will yield equal probabilities."

A first and seemingly trivial step towards this goal is simply to rethink how to enumerate alternatives – in particular, to rethink *what* is being enumerated[2]. Simply put, what is typically thought of as a *single result* – say, the n^{th} out of N possible results, with associated proba-

[1] The interested reader will find general discussion in Wallace ([3]), Greaves ([4]), Saunders ([5]) (add more refs), and references therein. Van Esch [6] argues that outcome counting (which he terms the "alternate projection postulate") is an internally consistent alternative to the Born rule. Weissman [7] and Hanson [8] have independently put forth specific proposals to implement outcome counting that may be compared and contrasted to the one in this paper.

[2] The "alternatives" being enumerated have been variously called "outcomes," "results," "superpositions," "worlds," "branches," etc. by different authors. Unfortunately there is no consistent use of terminology. This paper draws a distinction between "results" and "outcomes," whose meanings will hopefully be clear by context.

bility $|a_n|^2$ – should be equated with a *multitude of distinct outcomes*, the number of which is *proportional* to $|a_n|^2$. This approach attempts to justify the application of the Born rule to individual *results* by postulating the application of equiprobability to *outcomes*. In other words, equiprobability is the fundamental probability rule, applied at the fine-grained level where the distinction between outcomes is resolved. But at a more coarse-grained level – where one loses the distinction between outcomes and sees only the distinction between results – the Born rule applies.

The difficulty with this model – the reason it is trivial – is that it does not specify what an outcome actually *is*, or what makes one outcome *distinct* from another. Furthermore, there is a certain circularity to its reasoning. The Born rule measure is justified as a probability because its value is proportional to the number of outcomes associated with an experimental result; but conversely, the number of outcomes associated with a result is defined as its Born rule measure. A more satisfactory model, therefore, would avoid this circularity by achieving two goals. First, it would provide an *independent conception* of how outcomes are counted, one that does not make recourse to probability. Ideally, such a model would define "distinct" as *physically distinct*, would specify *how* they are physically distinct, and would specify how to enumerate them. Second, such a model would demonstrate that this independent method of outcome counting *reproduces the Born rule* at the coarse-grained level (i.e., at the level of results). Perhaps most importantly, the assumptions that make up the model would not include an explicit or implicit *assumption* of the Born rule.

The purpose of this paper, therefore, is to propose a model as discussed above that achieves these two goals, and so solves the quantitative problem in a non-trivial manner. Section 2 will define many of the terms to be used in this scheme. Section 3 will postulate their physical interpretation, applicable in the "low energy limit" [3]. The thrust of these sections will be to define a variable χ^1_{s,d_n} whose role is to tell us how to distinguish outcomes, as discussed above. In section 4, a very general quantum mechanical experiment will be posed: assuming a particle is emitted at location s (the source), calculate the relative probability of

[3] The concepts and terminology introduced in sections 2 and 3 borrow heavily from loop quantum gravity (LQG) [9]. It is tentatively suggested that the present work may be cast in the mold of LQG or one of its variants, although this is not explicitly demonstrated.

its detection at d_n (the n^{th} detector element). This experiment will be analyzed by counting the number of physically distinct states (outcomes) associated with the n^{th} result and postulating this value – which turns out to be $(\chi^1_{s,d_n})^2$ – to be the relative probability associated with that result. It will then be shown, using the Feynman path integral (FPI) technique [10], that this value is in proportion to its Born rule measure. Section 5 ends with a discussion of the significance of this result.

2 Mathematical preliminaries

2.1 Spin foams

A quantum mechanical spacetime will be represented using a *spin foam*, which is a (not necessarily connected) graph embedded in a 4-dimensional continuum W. The graph is built out of *nodes* connected by *links*. Each node is associated with a number C. Each link is associated with a number S, which we will interpret as the *action* of the link, with $\phi = e^{-iS/\hbar}$ being the *amplitude* of the link.[4] Links are oriented, in the sense that the action in one direction along a link is the negative of the action in the opposite direction.

A *fiber* is defined as an ordered set of links connecting one node to another, and a *loop* as a fiber that starts and ends at the same node. The action of a fiber is the sum of the individual actions of each link that make up the fiber. Action is assumed to be quantized, in the following sense: the action of any loop is an integer multiple of $\pi\hbar$:

$$\sum_j S_j = \pi\hbar m \tag{1}$$

with the sum being performed over all of the links in the loop, and m being any integer.[5]

2.2 Loop Complex

We will define and make use of several entities that can appear within spin foams. See Figure 1 for an illustration of the terms defined in this section.

[4] In LQG, C and S are interpreted as volume and area, respectively. This interpretation of C and S is perfectly compatible with the proposed theory; we merely stipulate that each link is associated with an area S that is proportional to a number $S' = \mu S$, μ being a conversion factor and a constant of the theory, and use S' in place of S as the action.

[5] One of the central results of LQG is that C and S are quantized [9], albeit not in exactly the same manner as postulated above.

A *unit loop of index* k is defined as a loop composed of k distinct nodes and k distinct links such that each link has action $\pi\hbar/k$ (so that the total action of the loop is $\pi\hbar$.) Each node in a unit loop is associated with the smallest quantum of volume, C, and is called a *unit node*.

A *loop complex of index* k is constructed as an ordered succession of unit loops, each with index k. The nodes in a unit loop are numbered $i = 1, 2, ..., k$, and the i^{th} node in one unit loop is connected by a fiber to the i^{th} node in the adjacent loop. Any fiber connecting the i^{th} node in one unit loop to the i^{th} node in a (not necessarily adjacent) unit loop will be referred to as a *path*. The index of a path is equal to the index of the loop complex. These paths may also be referred to as *spin foam paths* to distinguish them from the closely related but distinct Feynman particle paths (see below).

A loop complex may be open or closed. An *open loop complex* is one whose unit loops are arranged linearly. A *closed loop complex* is one that doubles back on itself, so that the unit loops are arranged as a cyclic group.

An *extended loop* is defined as a loop composed of two adjacent spin foam paths and the two links (components of unit loops) that connect them. The two unit loops may be, but are not necessarily, adjacent in the loop complex. We stipulate that the action around any extended loop is an *even* integer multiple of $\pi\hbar$:

$$\sum_j S_j = 2\pi\hbar m \tag{2}$$

with the sum being performed over all of the links in the loop, and m being any integer.

From the above, we may deduce that for $k > 1$, the sum of the amplitudes of the k paths connecting any two unit loops in a unit complex equals zero. Consider the extended loop in Figure 1A. Let S_i and S_{i+1} indicate the actions of the two spin foam paths in this extended loop. (Keep in mind that the action in one direction is the negative of the action in the reverse direction.) Applying equation 2 to the extended loop, we have:

$$\pi\hbar/k + S_i + \pi\hbar/k – S_{i+1} = 2\pi\hbar m \tag{3}$$

$$S_i – S_{i+1} = 2\pi\hbar m – 2\pi\hbar/k \tag{4}$$

This equation indicates that the amplitudes of two adjacent paths are $2\pi\hbar/k$ out of phase. This is depicted in Figure 1B, which shows that the amplitudes of the k paths are equidistributed on the unit circle. We arrive at the conclusion that the sum of the amplitudes of the k paths of the complex equals zero. This statement is not true for $k = 1$, in which case there is only a single path, whose amplitude is not constrained by the above considerations.

Define D_l as the *unit loop density* as follows. Consider an (arbitrarily small) 4-region A in a foam. Designate the total number of unit loops in A as u_l, and the total number of unit nodes in A as u_n. The unit loop density at A is defined as the ratio $D_l = u_l/u_n$.

3 Physical interpretation in the low energy approximation

Each spin foam is a quantum mechanical spacetime. For the description of any given experiment, we will require not one, but rather an *ensemble* of spin foams which exist in *superposition* to one another. For the physical interpretation of a given foam, we will assume a "low energy approximation," which consists of the following series of assumptions.

3.1 The existence of a background manifold

The first of these assumptions is the existence of a 4-dimensional manifold M that serves as a background space for the theory. (In the non-relativistic case, M is simply flat spacetime, i.e. $\mathbb{R}^4$.) M may be thought of (loosely) as a sort of blank movie screen on which each foam in the ensemble may be individually projected. This process of "projection" from a foam onto M is achieved through a mapping function, ζ, which inputs one foam (or subregions of a foam) and outputs an *image* of the foam (or the subregion) on M. Thus, the images of nodes and fibers in a foam will be points and curves in M. In addition, ζ will output the unit loop density D_l as well as a distribution of mass-energy (particles, potential wells, etc.) over M for each foam in the ensemble.

An ensemble of foams gives rise to an ensemble of images, and these images will agree on some details (especially macroscopic ones), and disagree on others (especially microscopic ones). Any detail that fits under the rubric of "experimental setup" (see below) must be a common feature of *every* image in the ensemble.

3.2 The images of spin foam paths are Feynman particle paths

For any given spin foam, we will assume that the set of images of all spin foam paths is in one-to-one correspondence to the set of all possible paths in M, as generated by the Feynman path integral (see below).

Furthermore, we assume that the action of a given Feynman path in M, as determined by the FPI, is equal to the action of the corresponding spin foam path. The distribution of actions (and hence amplitudes) of the set of "all possible paths" is determined uniquely from the experimental setup via the FPI technique (see below) and hence must be a feature that every foam in the ensemble agrees upon.

3.3 Loop density is minimized

The unit loop density D_l of an (arbitrarily small) spacetime region A is, in large part, a function of the ultra-local structure of the spin foam in A; in particular, of the precise manner with which different spin foam paths are "coassociated" into loop complexes. This is illustrated in Figure 2B. Elements of two foams are depicted giving rise to identical images consisting of two (arbitrarily small) 4-regions, A and B, and four paths that connect them. The two foams agree that the actions of these four paths (mod $2\pi\hbar$) are 0, $\frac{1}{2}\pi\hbar$, $\pi\hbar$, and $\frac{3}{2}\pi\hbar$. The difference between the two foams is that the four paths are connected into two index $k = 2$ loop complexes in the upper panel foam, as opposed to one index $k = 4$ complex in the lower panel foam. Such ultra-local variability can lead to wide variation in D_l from one foam to the next.

In the low energy approximation, unit loop density D_l is assumed to take the minimum possible value at each point in M.[6] By this rule, the foam in the upper panel of Figure 2B is excluded from the ensemble. Since the solution to D_l^{min} over M depends only upon the set of all possible paths and their actions, and each of these is prescribed uniquely by the experimental setup, then the solution to D_l^{min} over M is likewise unique for a given experimental setup.

This still allows a large degree of microscopic variability from one spin foam to another in the ensemble. For example, the difference between one foam and another may be nothing more than a "swap" of two spin foam paths between one loop complex to another, which is permissible if they are of identical action (mod $2\pi\hbar$).

3.4 Particle model

Our simple model of a particle in a spin foam will be to stipulate the following: for each particle, there corresponds a single closed loop complex of index $k = 1$, the significance of which is that its associated spin

[6] If we interpret S and C as action and volume, respectively, then the minimization of unit loop density D_l could be viewed as a least action principle.

foam paths trace the trajectory of the particle through 4-space. In other words, if a particle is observed in the (arbitrarily small) spacetime region A in a spin foam, and is additionally observed in another spacetime location B, then there must exist an associated closed loop complex of index $k = 1$, with one unit loop in A and the other in B. This will, in turn, mean that out of all the particle paths connecting A to B, two of them are "special" in the sense that they correspond to the two ways around the closed loop complex associated with the particle. Furthermore, these two "special" spin foam paths must be of index $k = 1$. These two paths through 4-space need not (necessarily) be in close proximity to one another between A and B.[7] Of course, there will typically be additional $k = 1$ spin foam paths connecting A to B that are not associated with this particular particle. These are either not associated with particle trajectories, or are associated with the trajectories of other (unrelated) particles. In addition, there will also be spin foam paths of $k > 1$ connecting A to B that are also not associated with the particle.

3.5 Definition of $\chi^k_{A,B}$

Given a particle that traverses the (arbitrarily small) spacetime regions A and B in a spin foam as discussed above, we imagine there to be a *very large number* of unit loops in A, and likewise in B, ranging over all indices, and giving rise to a *very large number* of spin foam paths stretching between A and B. (Recall that there must be one spin foam path for each of the "all possible paths" generated by the FPI.) Define $\chi^k_{A,B}$ as the number of spin foam paths of index k connecting region A and region B.

4 Experiment

We will consider a very general quantum mechanical experiment: given a particle that is observed within the (arbitrarily small) spacetime region s (the source) in M, calculate the relative probability that it will be detected within the (arbitrarily small) spacetime region d_n (in which is located the n^{th} of N detector elements). This can be used to model (among other things) the 2-slit experiment, which Feynman has argued contains the "essential mystery" of QM.

[7] It would be interesting to consider whether one direction around this closed loop might correspond to a particle, with the other direction corresponding to an antiparticle.

Let us review how the above scenario is analyzed by the Feynman path integral technique. This is performed in the background M. The first step is to enumerate the set of "all possible paths" from the source s to the detector d_n. Each path has an associated action S and amplitude $\phi = e^{-iS/\hbar}$. Calculation of the action requires knowledge of the potential as a function of spacetime location, so it is assumed that this is included in the experimental setup. The wavefunction a_n is calculated as the sum of the amplitudes ϕ of every path from s to d_n: $a_n = \sum \phi$.

Each foam in the ensemble, therefore, must agree on the following details: a particle emitted from a source s, an array of N detector elements d_n, the potential, and D_l. There are, however, several important details that can vary from foam to foam. These include which detector detects the particle; the path(s!) that the particle took from source to detector; and, as discussed above, the ultra-local details underlying which spin foam paths are coassociated with which.

4.1 There are $(\chi^1_{s,d_n})^2$ outcomes associated with the n^{th} result

Recall that for a given spin foam, the trajectory of a particle through 4-space is traced by two uniquely associated spin foam paths that are components of a closed loop complex with unity index ($k = 1$). For any given spin foam, the total number of index $k = 1$ spin foam paths from s to d_n equals, by definition, χ^1_{s,d_n}. Within any particular spin foam, if the particle in question is observed in the region d_n, then *two and only two* of these χ^1_{s,d_n} spin foam paths correspond to the particle under observation, with the remaining $k = 1$ paths being irrelevant to the particle in question.

So let us postulate that the process of observation of the particle at the detector corresponds to identification of these two out of χ^1_{s,d_n} spin foam paths. Note that the number of ways to pick two out of χ^1_{s,d_n} paths is "2 choose χ^1_{s,d_n}" i.e. $\frac{1}{2}(\chi^1_{s,d_n})(\chi^1_{s,d_n} - 1)$, which is proportional to $(\chi^1_{s,d_n})^2$ for large χ^1_{s,d_n}. Therefore, an observer identifying these two paths will identify one out of approximately $(\chi^1_{s,d_n})^2$ distinct outcomes. We might postulate that each one of these $(\chi^1_{s,d_n})^2$ outcomes corresponds to *one distinct physical state* of the observer, induced by the process of observation.

So let us postulate that this is the physical variable that we have been looking for that makes one outcome distinct from another. According to the central tenet of this paper, the probability of a given result is proportional to the number of outcomes associated with that result. Therefore, the number of outcomes associated with the n^{th} result is $(\chi^1_{s,d_n})^2$. Therefore, we predict the probability of detection at detector element d_n to be proportional to $(\chi^1_{s,d_n})^2$.

4.2 The minimization of loop density implies $\chi^1_{s,d_n} \sim |a_n|$

As discussed above, the minimization of D_l implies that as many unit nodes – and hence, as many spin foam paths – as possible will be coassociated into loop complices of high index; and conversely, that the number of paths with index $k = 1$ will be minimized. Thus, the minimization of D_l implies the minimization of χ^1_{s,d_n}. What can we say about the lowest possible value of χ^1_{s,d_n}?

Consider the constraint that k spin foam paths can only be coassociated into a single loop complex if their amplitudes are equidistributed on the unit circle. Consider a subset of all of the paths from s to d_n, with p paths in the subset. If their amplitudes are distributed equally on the unit circle, we will call this an *equidistributed* subset, and will note that these paths may be coassociated into a single loop complex of index $k = p$ (Figure 2A, left). We also note that the sum of the amplitudes of these p paths equals zero. Suppose now that we have a subset of p paths whose amplitudes sum to zero, but which are not necessarily equidistributed (Figure 2A, middle). We will refer to this as a *symmetrically distributed* subset of paths. If the sum of the amplitudes is not zero, we call this an *asymmetrically distributed* subset of paths (Figure 2A, right). We remark that it is always possible for a symmetrically distributed subset of paths to have the characteristic that each and every path in the subset is of index $k > 1$. On the other hand, for an asymmetrically distributed subset, there will always be at least one path with index $k = 1$.

We further remark that for any set of all paths from s to d_n, it is possible to divide this into two subsets, a symmetric and an asymmetric one. It should be apparent that if the number of paths in the symmetric subset is maximized, with the number of paths in the asymmetric subset therefore minimized, then *every* path in the asymmetric subset will be of index $k = 1$, and furthermore will be of the *same phase* [8]. Since they are of the same phase, then the number of paths in the asymmetric subset

[8] This analysis assumes that we do *not* apply the semiclassical approximation to the FPI.

(χ^1_{s,d_n}) will be proportional to the absolute value of the sum of their phases $(|a_n|)$. Therefore, the low energy approximation implies that:

$$\chi^1_{s,d_n} \sim |a_n| \tag{5}$$

Squaring yields:

$$(\chi^1_{s,d_n})^2 \sim |a_n|^2 \tag{6}$$

Note that the left hand side of the above equation is the relative probability of detection at the n^{th} detector element, as predicted by the proposed model (above). The right hand side of the above equation is, of course, the same probability, as predicted by the standard Born rule.

Therefore we may conclude what we set out to prove, which is that the proposed scheme makes the same prediction as the Born rule, and hence of quantum mechanics in general. It is noteworthy that the list of assumptions that lead to the left hand side of the above equation does *not* include the Born rule, either overtly or in disguised form. It may be claimed that the proposed model therefore represents at least the sketch of a derivation of the Born rule from independent assumptions.

4.3 Discussion

The primary motivation for this scheme is the philosophical conviction that outcome counting is the only probability rule that fits the ontological framework of the MWI. The purpose of this paper has therefore been to propose a solution to the quantitative problem of the MWI in which probability is fundamentally governed at the fine-grained level by outcome counting. Quantum statistics is then recovered by demonstrating that the Born rule applies at a more coarse-grained level. To avoid circular reasoning, the number of outcomes is determined by counting the number $(\chi^1_{s,d_n})^2$ of *distinct physical states* associated with the process of observation.

The secondary motivation for the development of this scheme is the prospect that it could be a facet of a larger theory of quantum gravity (QG). Many of the concepts introduced in section 2 are borrowed, albeit loosely, from loop quantum gravity (LQG), and it is tentatively suggested that the proposed scheme could be made to fit the framework of LQG or some related theory of QG. If such is the case, then the true test of the value of the proposed scheme will be whether its incorporation is an aide or an obstacle to the further development of an overall theory.

So how might the proposed scheme be of such aide? In principle, its role would be to refine the search for a theory of QG by pointing out certain mathematical elements that should be present. This will require identifying exactly which elements of the proposed model are essential, and which are extraneous. There are likely many small variations that could be made to the proposed scheme that would preserve its essential structure. The basic idea is to start with a superposition of spacetimes, and to assume that each spacetime contains its own representation of the set of "all possible" Feynman paths. A scheme is then established which allows for symmetric subsets of paths to "cancel each other out" – meaning that we can ignore them – leaving a collection of leftover paths whose number χ must be proportional to the wavefunction. Finally it is established that detection of the particle is concomitant with choosing two of these leftover paths, so that the number of distinct physical states induced by the observation process is "2 choose χ" $\sim \chi^2$. It is not until this last step that a probability rule is implemented, which is that each of these χ^2 states is *equally likely.* Most importantly, the essential structure of this scheme is designed so that *the Born rule is not assumed*, but rather *derived*.

At the very least, the proposed theory may be considered as an *existence proof* demonstrating the possibility that a theory of QG can be founded on a fundamentally classical notion of probability – outcome counting – and yet still lead in the approximation to quantum statistics.

References

[1] Hugh Everett. Relative state formulation of quantum mechanics. *Reviews of Modern Physics*, 29:454–462, 1957.

[2] Neill Graham. The measurement of relative frequency. *The Many-Worlds Interpretation of Quantum Mechanics*, pages 229–252, 1973.

[3] David Wallace. Quantum probability and decision theory, revisited. *Studies in the History and Philosophy of Modern Physics*, 34:415–438, 2003.

[4] Hilary Greaves. Understanding deutsch's probability in a deterministic multiverse. *Studies in the History and Philosophy of Modern Physics*, pages http://philsci–archive.pitt.edu/archive/00001742/, 2004.

[5] Simon Saunders. Time, quantum mechanics, and probability. *Synthese*, 114:373–404, 1998.

[6] Patrick Van Esch. On the everett programme and the born rule. *quant-ph/0505059*, 2005.

[7] M. B. Weissman. Emergent measure-dependent probabilities from modified quantum dynamnics without state-vector reduction. *Foundations of Physics Letters*, 12:407–426, 1999.

[8] Robin D. Hanson. When worlds collide: Quantum probability from observer selection? *Foundations of Physics*, 33:1129 – 1150, 2003.

[9] H. Nicolai and Kasper Peeters. Loop and spin foam quantum gravity: a brief guide for beginners. *hep-th/0601129*, 2006.

[10] R.P. Feynman and A.R. Hibbs. *Quantum Mechanics and Path Integrals*. McGraw-Hill, Boston, 1965.

(Manuscrit reçu le date)

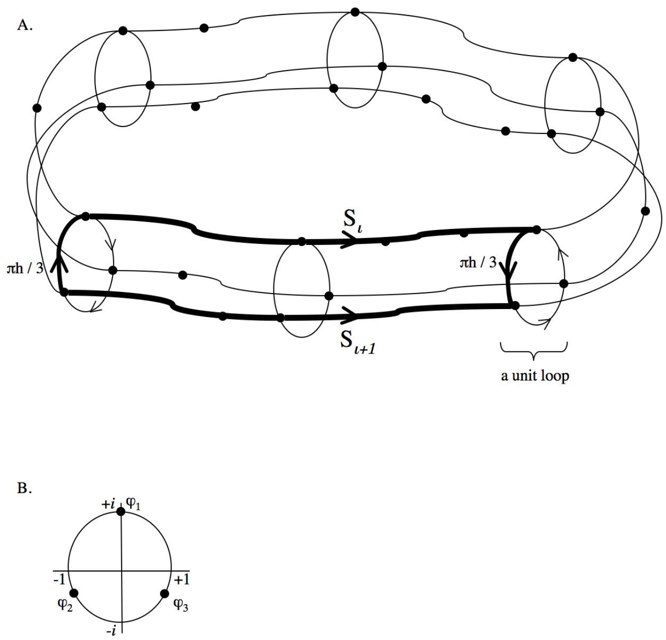

Figure 1: A. A closed loop complex of index $k = 3$, composed of six unit loops. Small filled circles are nodes, and lines connecting them are links. An extended loop is shown in bold; this contains two spin foam paths with actions S_i and S_{i+1}, as well as two links (components of unit loops) each with action $\pi\hbar/3$, as indicated. The actions of links are oriented (indicated by arrows). B. The amplitudes $\phi = e^{-iS/\hbar}$ of three adjacent spin foam paths of index $k = 3$ are plotted on the complex plane, illustrating that they are equidistributed on the unit circle and that they sum to zero.

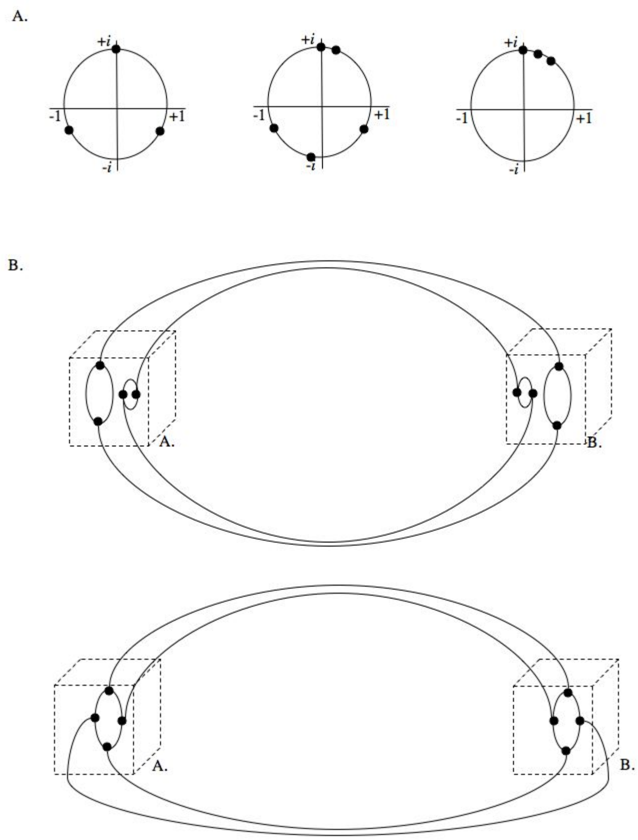

Figure 2: A. Amplitudes of spin foam paths are plotted on the complex plane as in Figure 1B, demonstrating an *equidistributed* subset of 3 paths (left), *symmetrically* distributed subset of 5 paths (middle) and *asymmetrically* distributed subset of 3 paths (right). B. Values of $\chi^k_{A,B}$ vary from one foam to the next as a result of differences in the microscopic spin foam structure within the spacetime regions A and B (indicated by boxes). The same four spin foam paths may be composed into either two $k = 2$ closed loop complexes (upper panel) or one $k = 4$ complex (lower panel). The principle of D_l minimization favors the lower foam over the upper one.