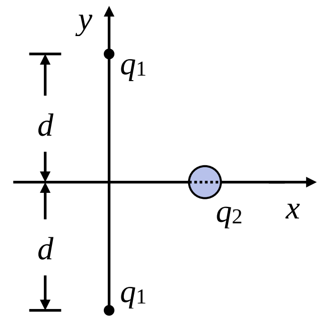
1. COULOMB'S LAW

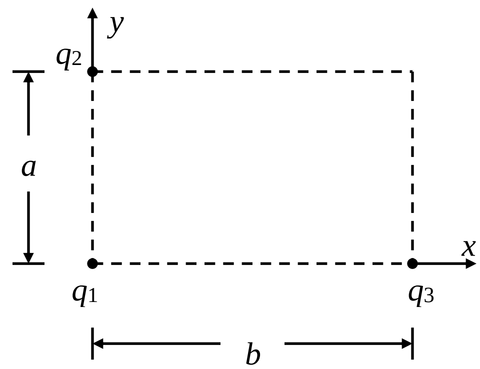
Two identical particles, each having charge q_1 = 2.50 μC, are fixed in space on either side of the y-axis at distance d = 20.0 cm. A plastic bead with charge q_2 = -3.00 μC and mass m = 8.00 g is constrained to move on a rod along the x-axis as shown in the figure. The bead is released from rest at x = 0.1 cm.



1.1 Show that if x is very small compared with d, the motion of q_2 is simple harmonic along the perpendicular bisector and determine the period of the motion. [0.137 s]

1.2 How fast will q_2 be moving when it is at the midpoint between the two fixed charges? [4.59 cm/s]

Three charges are fixed at the corners of a rectangle of base b = 5.00 cm and height a = 3.00 cm as shown. Charge q_1 = -4.50 μC is at the origin, charge q_2 = 6.00 μC is at (0, a) and unknown charge q_3 is at (b, 0). It is observed that the net force on charge q_1 is along the diagonal of the rectangle and points towards the empty corner.



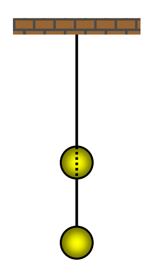
1.3 Find the magnitude and sign of q_3. [+27.8 μC]

1.4 At what coordinates in the xy plane must a fourth charge, identical to q_3, be placed so that the net force on q_1 becomes zero? [(-3.97 cm, -2.38 cm)]

Three identical beads mass m = 0.280 kg are tied to strings of length L = 1.00 m and suspended from a common point at the ceiling. All beads bear identical charge q. At equilibrium, the beads form an equilateral triangle with sides of length a = 25.0 cm.

1.5 Find the magnitude of the charge q. [1.27 μC]

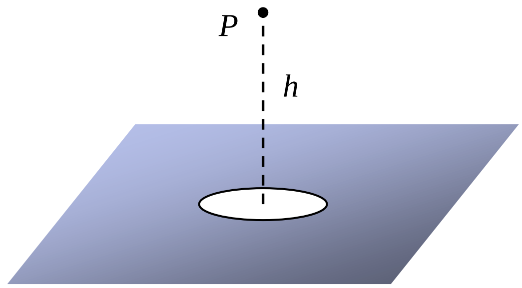
Two identical beads with identical charges q = +4.80 μC and mass m = 80.0 g are on an insulating string that hangs straight down from the ceiling as shown in the figure. The lower bead is fixed in place, but the second bead is free to slide without friction on the string. The beads are at rest in equilibrium.



1.6 Find the distance between the beads. [0.514 m]

1.7 Find the tension in the string. [1.57 N]

2. ELECTRIC FIELDS AND GAUSS'S LAW



Point P is directly above the center of the hole at distance h = 20.0 cm as shown in the figure. The hole has diameter D = 10.0 cm and is cut out of an infinite sheet of charge with charge distribution σ = 1.30 μC/m^2.

2.1 Find the electric field at point P. [7.13×10^4 N/C, straight up]

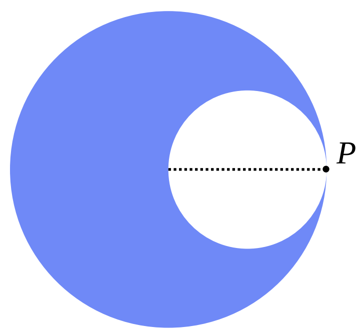
A solid non-conducting sphere has volume charge distribution given by $\rho(r) = (\beta/r)\sin[\pi r/(2R)]$. Here R = 24.0 cm is the radius of the sphere and β = 1.24 μC /m^2.

2.2 Find the magnitude of the electric field at r = 12.0 cm. [3.33×10^4 N/C]

2.3 Find the magnitude of the electric field at r = 36.0 cm. [2.74×10^4 N/C]

Charge q is fixed at the origin and charge $4q$ is fixed on the x-axis at x = 12.0 cm.

2.4 Find the point on the y-axis where the net electric field makes an angle of 45° with respect to the y-axis. [8.87 cm]



A solid sphere of radius R_1 = 40 cm has a spherical cavity of radius R_2 = 20 cm. The center of the cavity is at d = 20 cm from the center of the sphere (see figure). The sphere has total charge Q = 2.40 μC uniformly distributed over its volume. The dotted line indicates the radius of the sphere that passes through the center of the cavity.

2.5 Find the electric field at point P. [8.44×10^4 N/C, to the right]

2.6 Find the electric field at the center of the sphere. [6.75×10^4 N/C, to the right]

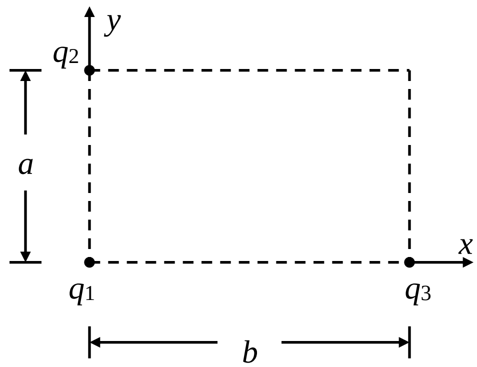
A semi-infinite line of charge starts at the origin and extends along the positive x-axis. It carries linear charge density λ = +1.80 μC/m.

2.7 Find the electric field at point (12.0 cm, 24.0 cm). [-6.04×10^4 N/C, 9.77×10^4 N/C]

3. ELECTRIC POTENTIAL ENERGY AND ELECTRIC POTENTIAL

Note: In all problems below, the potential is taken to be zero at infinity.

Three charges are fixed at the corners of a rectangle of base b = 5.00 cm and height a = 3.00 cm as shown. Charge q_1 = -4.50 μC is at the origin, charge q_2 = 6.00 μC is at (0, a) and unknown charge q_3 is at (b, 0). It is observed that the electrostatic potential at the empty corner is zero..



3.1 Find the magnitude and sign of q_3. [-1.28 μC]

3.2 Find the total electrostatic potential energy of the three-charge assembly. [-8.25 J]

3.3 Find the work done by the electrostatic forces when charge q_2 is moved from its current location to the empty corner. [-2.81 J]

A solid non-conducting sphere has volume charge distribution given by $\rho(r) = \beta r$. The radius of the sphere is R = 20.0 cm and β = 3.60 μC /m^4.

3.4 Find the electric potential at r = 24.0 cm. [679 V]

3.5 Find the electric potential at r = 12.0 cm. [1.03 kV]

A proton has mass $m_P = 1.67\times10^{-27}$ kg and charge $e = +1.60\times10^{-19}$ C. An alpha particle has mass $4m_P$ and charge $2e$. A proton and an alpha particle are initially very far from each other. They travel straight towards each other with equal speed v_0 = 350 m/s.

3.6 Find the distance of closest approach. [1.41×10^{-6} m]

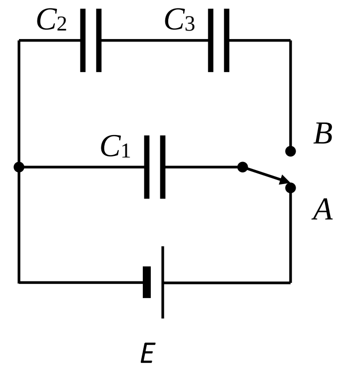
3.7 Find the speed of the alpha when the particles are very far from each other again. [70.0 m/s]

A thin line of charge is aligned along the positive x-axis from the origin to $x = L$, where L = 20.0 cm. The linear charge distribution is non-uniform and given by $\lambda = +\beta x$ where β = 6.50 μC/m^2.

3.8 Find the electric potential at point P = (0.0 cm, 10.0 cm). [7.23 kV]

3.9 Find the y-component of the electric field at point P = (0.0 cm, 10.0 cm). [3.23×10^4 V/m]

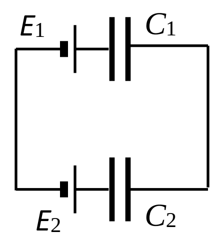
4. CAPACITORS



Three capacitors, C_1 = 40.0 pF, C_2 = 30.0 pF and C_3 = 60.0 PF are connected as shown. Initially, the switch is at position A and capacitor C_1 is fully charged by the battery (E = 12.0 Volts) while C_2 and C_3 are initially uncharged. The switch is thrown to B and the charge on C_1 redistributes itself.

4.1 Find the voltage across C_1. [8.00 Volts]

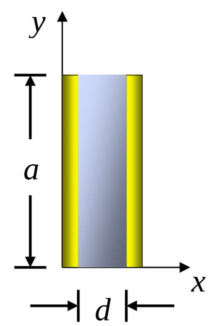
4.2 Find the voltage cross C_3. [2.67 Volts]



Two capacitors, C_1 = 20.0 pF and C_2 = 30.0 pF are connected as shown to two batteries, E_1 = 12.0 V and E_2 = 18.0 V.

4.3 Find the charge stored in C_2. [72.0 pC]

4.4 Find the voltage across C_1. [3.60 V]

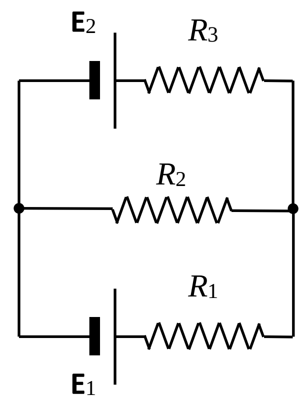


A parallel plate capacitor is formed by two square metal plates of side a = 40.0 cm separated by distance d = 5.00 cm. A dielectric slab of non-uniform dielectric constant is inserted snugly between the plates. In the following questions the two constants have values κ_0 = 2.60 and α = 1.20 m^{-1}.

4.5 Find the capacitance if the dielectric constant depends on the *horizontal* distance x from the left plate according to $\kappa(x) = \kappa_0 + \alpha x$. [74.4 pF]

4.6 Find the capacitance if the dielectric constant depends on the *vertical* distance y from the bottom of the plates according to $\kappa(y) = \kappa_0 + \alpha y$. [80.3 pF]

5. RESISTORS AND CIRCUITS

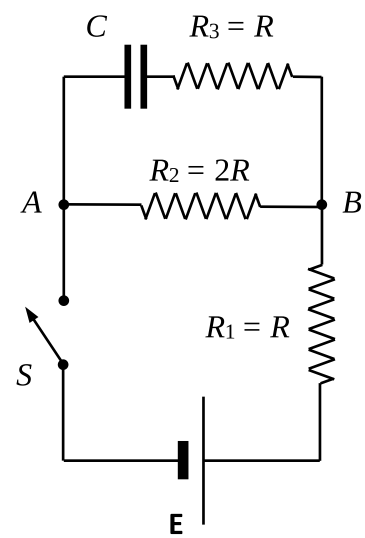


Three resistors, $R_1 = 20.0\ \Omega$, $R_2 = 30.0\ \Omega$ and $R_3 = 40.0\ \Omega$ are connected as shown to two batteries, $\mathsf{E}_1 = 12.0$ V and $\mathsf{E}_2 = 18.0$ V.

5.1 Find the current in R_1. [15/130 A]

5.2 Find the current in R_2. [42/130 A]

5.3 Find the current in R_3. [27/130 A]



In the circuit shown, $R = 10{,}000\ \Omega$, $C = 2.40$ nF and the battery's emf is $\mathsf{E} = 12.0$ V. The switch is initially open and the capacitor is uncharged. At time $t = 0$ the switch is closed. At $t = t_1$ later, the potential difference between points A and B is 2.00 V.

5.4 Find the current provided by the battery at time $t = t_1$. [1.00 mA]

5.5 Find the charge on the capacitor at time $t = t_1$. [7.20 nC]

5.6 Find the time constant τ of the circuit. [40.0 µs]

5.7 Find time t_1. [18.8 µs]

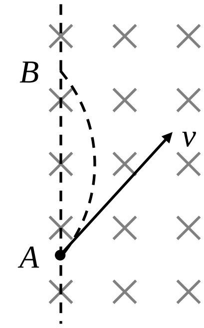
5.8 Find the total energy dissipated as heat in resistor R_3 over the time interval from $t = 0$ to $t = t_1$. [2.81×10^{-8} J]

A piece of aluminum is in the shape of a tapered rod of circular cross section and length $L = 1.20$ m. The taper is constant and such that the radius is $r_1 = 1.20$ cm at one end and $r_2 = 2.40$ cm at the other. The resistivity of aluminum is $\rho_{Al} = 2.65\times10^{-8}\ \Omega\cdot$m.

5.9 Find the resistance between the two ends. [$3.51\times10^{-5}\ \Omega$]

6. PARTICLES, MAGNETIC FIELDS AND CURRENTS

A uniform magnetic field B = 0.620 T points into the page. A proton enters the region at point A with velocity perpendicular to the magnetic field lines but at angle θ = 54.0° with respect to the edge of the field (see figure). The initial speed of the proton is $v = 6.80 \times 10^5$ m/s. The proton exits the field at point B.

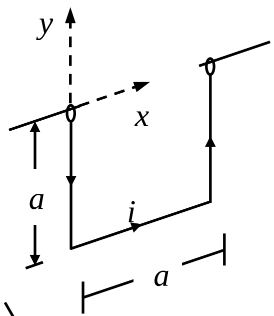


6.1 Find distance AB. [1.85 cm]

6.2 Find the time the proton spends in the field region. [1.59×10^{-8} s]

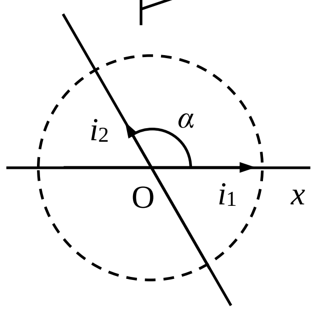
6.3 Find the average force exerted by the field on the proton. [8.41×10^{-14} N]

A U-shaped wire of equal sides, a = 30.0 cm, and total mass 60.0 g is suspended from a horizontal wire (see figure). The wire can swing freely like a pendulum. Current i = 1.80 A is fed into the wire. A uniform magnetic field B = 0.240 T is directed in the y-direction, opposite to the acceleration of gravity.



6.4 Find the equilibrium angle that the plane of the loop forms with respect to the vertical direction (y-axis). [47.8°]

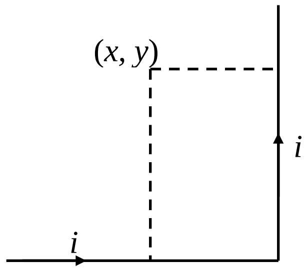
Two very long wires carrying currents i_1 = 12.0 A and i_2 = 28.0 A are in the plane of the page and cross at the origin (point O) as shown. The wires are insulated so their currents are independent of one another. Angle α = 120°.



6.5 On the dotted circle, find the smallest angle with respect to the positive x-axis at which the net magnetic field is zero. [25.3°]

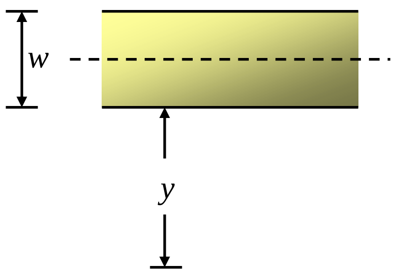
A very long wire carrying current i = 16.0 A is bent into a 90° angle as shown. The origin is at the corner.

6.6 Find the net magnetic field at x = 48 cm, y = 64 cm. [10.0 μT]



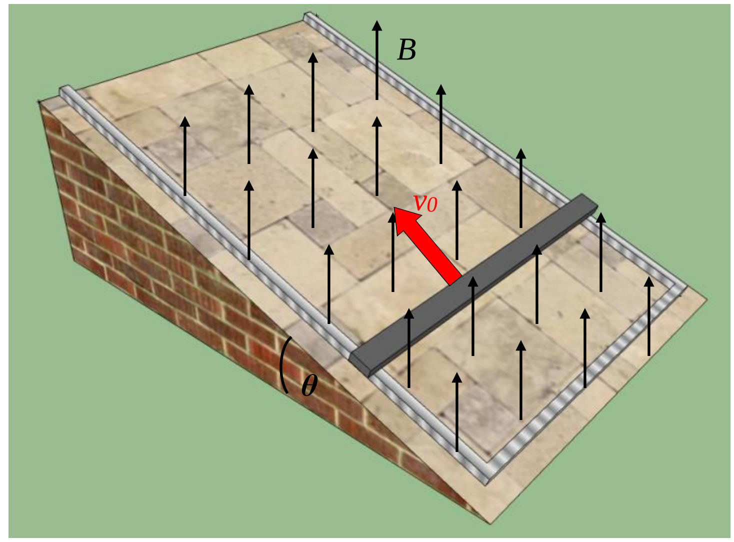
A very long metal strip of width w = 4.00 cm is in the plane of the page and carries total current i = 3.80 A.

6.7 Find the magnitude of the magnetic field at a point in the plane of the page at distance y = 6.20 cm from the edge of the strip. [9.46 μT]



7. MAGNETIC FLUX, INDUCTION AND FARADAY'S LAW

A rod with length L = 40.0 cm, mass m = 250 g and resistance R = 2.00 Ω slides without friction on parallel conducting rails of negligible resistance (see figure). The rails are connected at the bottom as shown, forming a conducting loop with the rod as the top member. The plane of the rails makes an angle θ = 37° with the horizontal and a uniform vertical magnetic field B = 1.60 T exists throughout the region. The rail is given an initial velocity v_0 = 22.0 m/s up the incline, it reaches maximum distance and returns.



7.1 Find the time it takes the rod to reach maximum distance. [1.88 s]

7.2 Find the maximum distance. [16.6 m]

7.3 Find the total energy dissipated as heat in the rod from the starting point until it reaches maximum distance. [45.1 J]

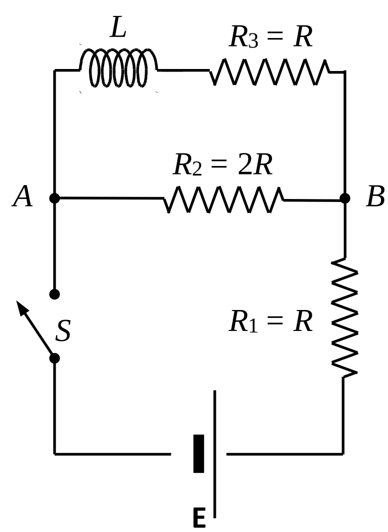
In the circuit shown, R = 10.0 Ω, L = 6.40 mH and the battery's emf is E = 12.0 V. The switch is initially open. At time t = 0 the switch is closed. At $t = t_1$ later, the potential difference between points A and B is 2.00 V.

7.4 Find the current provided by the battery at time $t = t_1$. [1.00 A]

7.5 Find the voltage across the inductor at time $t = t_1$. [3.00 V]

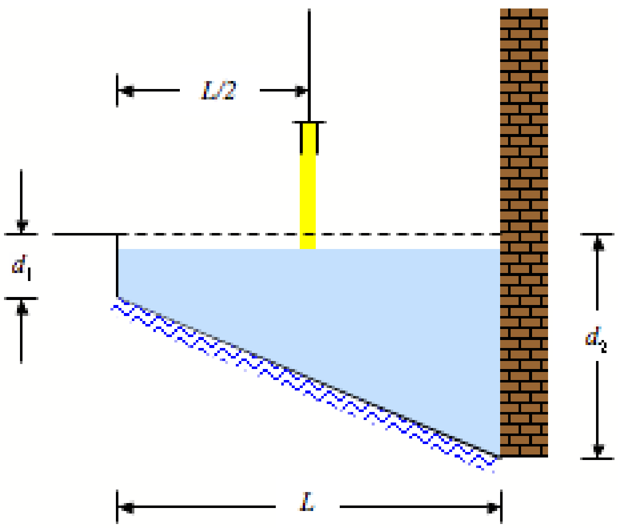
7.6 Find the time constant τ of the circuit. [0.384 ms]

7.7 Find time t_1. [0.377 ms]



8. REFLECTION AND REFRACTION

A rectangular, belowground indoor pool has a sloping bottom. The shallowest point is d_1 = 2.00 m below ground and the deepest point is d_2 = 6.00 m below ground. The bottom is reflecting and the length of the pool is L = 15.0 m. A spotlight hangs over the pool at the midpoint, i.e. 7.50 m from either end (see figure drawn not to scale). Initially the pool is empty.



8.1 Find how far above ground level (dotted line) the reflected light appears on the vertical wall rising over the deepest end. [9.06 m]

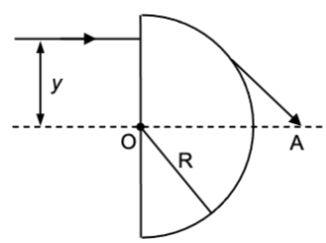
8.2 The pool is now filled with water (n = 4/3) to level h = 0.500 m below ground. Find the new position of the reflected light. [5.68 m]

A concave mirror of focal length f = 1.20 m lies on the floor. A small object is above the mirror on the optical axis. It is released from rest at distance 3.60 m above the mirror.

8.3 Find the position of the image at time t = 0.500 s after release. [2.43 m]

8.4 Find the speed of the image at time t = 0.500 s after release. [5.11 m/s]

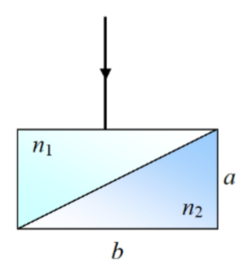
The figure shows the cross section of a plastic half-disk of index of refraction n = 1.55 and radius R = 12.0 cm. A ray is incident parallel to the symmetry axis and exits out the other side.



8.5 Find distance y for which distance OA has the smallest value (see figure). [7.74 cm]

8.6 Find the smallest value of distance OA. [15.7 cm]

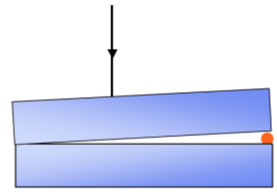
Two plastic wedges of identical geometry but different indices of refraction (n_1 = 1.75 and n_2 = 1.45) are placed with their hypotenuses together so that they form a block (see figure). Side a = 36.0 cm. Light is incident perpendicularly on the top wedge and is refracted at the diagonal interface.



8.7 Find the smallest value of b that will allow the light to enter the bottom wedge and then be internally reflected at side b. [25.2 cm]

9. INTERFERENCE

An "air" wedge is formed by two glass plates of length L = 4.00 cm touching at one end and separated by a copper wire of diameter d = 0.05 mm at the other. A monochromatic ray of light is incident at near normal incidence and bright fringes are formed. The wavelength of the light is 595 nm.

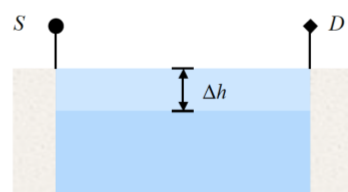


9.1 Find the distance from the touching edge to the 44th bright fringe. [1.03 cm]

A second ray of light consisting of monochromatic wavelength of 665 nm, is incident on the air wedge of the previous problem. It is found that, at the *n*th bright fringe from the vertex of the air wedge for the 665 nm wave, a bright fringe for the 595 nm wave is also formed.

9.2 Determine the smallest value of n. [10]

Source S and detector D are separated by distance SD = 120.0 m across a lagoon. The source emits radio waves of wavelength λ = 33.4 cm. At high tide the water level is at h = 8.40 m below the line joining the source and detector and a very strong signal is detected, but as the water recedes, the signal weakens and reaches zero at the low tide mark.



9.3 Find the change Δh of the water level between the high and low tide marks. [58.3 cm]

A double slit of is illuminated simultaneously with orange light of wavelength 600 nm and light of an unknown wavelength. The slit separation is 5.00×10^{-2} mm. The m = 5 bright fringe of the unknown wavelength overlaps the m = 4 bright orange fringe on a screen that is 2.40 m away from the double slit.

9.4 Find the unknown wavelength. [480 nm]

9.5 Find the distance on the screen between the m = 4 orange bright fringe and the m = 4 fringe of the unknown wavelength. [2.30 cm]