

**MATH
PHYSICS
ENGINEERING**

HOBBYIST

**Air cannon fundamentals
&
Optimized barrel length**



OBJECTIVE

To create a hypothetical homemade cannon out of appropriately rated PVC pipes and components.

- Projectiles will be potato like in terms of mass, size and shape.
- Achieve a projectile exiting velocity between 75-100 m/s (246-328 ft/s).
- Maximum pressure will be 820 kPa (120 PSI) - Check components rating for the build!.
- Barrel length and pressure chamber size will be optimized.



CANNON' SOURCE OF ENERGY



Combustion	
Pros	Cons
<ul style="list-style-type: none">• BOOM is cool• No valve is needed	<ul style="list-style-type: none">• Unreliable stoichiometry• Availability of fuel

Pressurized air	
Pros	Cons
<ul style="list-style-type: none">• Availability of fuel• Reliable pressure	<ul style="list-style-type: none">• Pressurizing air takes time• Opening time of the valve is key

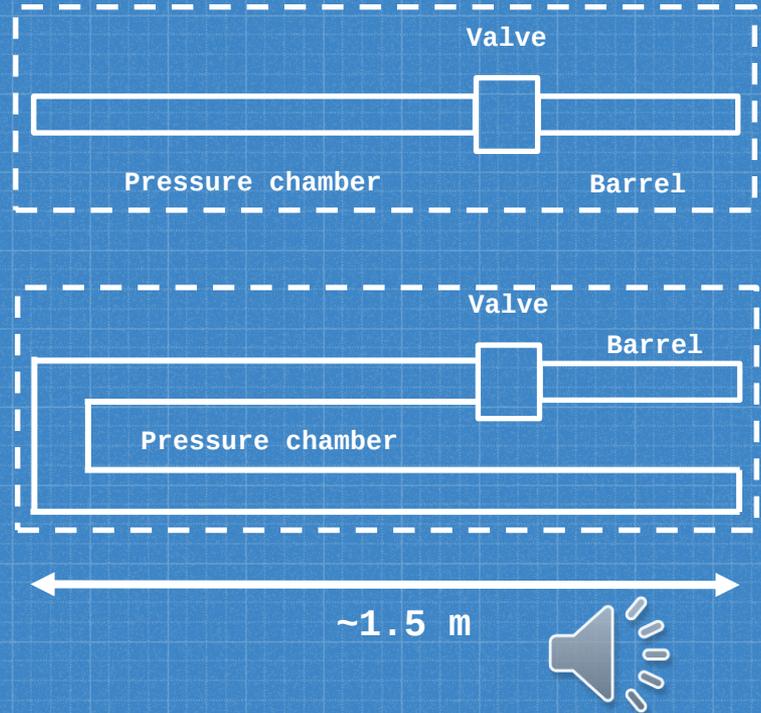
Dry ice	
Pros	Cons
<ul style="list-style-type: none">• Dry ice is cool <small>(get it?)</small>	<ul style="list-style-type: none">• Availability of fuel• Opening time of the valve is key

Combustion + Pressurized air	
Pros	Cons
<ul style="list-style-type: none">• BIG BOOM is cooler than BOOM	<ul style="list-style-type: none">• Outright dangerous and scary• Stronger materials for extra safety



Overall shape for the pneumatic cannon

- Total length ~1.5 m (590 in).
- Barrel diameter 50 mm (1.96 in).
- Ergonomic to be aimed from the hip.
- Pressure chamber to barrel volume ratio is TBD.



Sizing the pressure chamber and barrel length

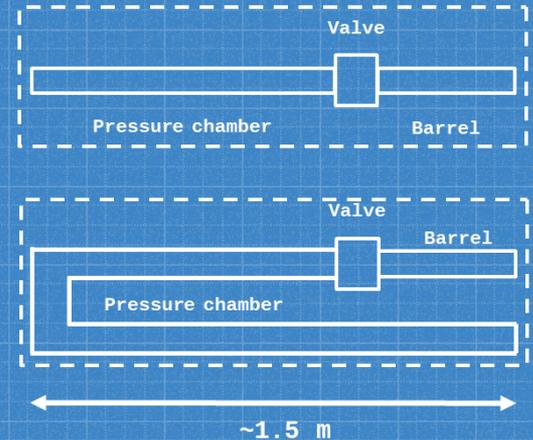
Problem description and simplifications

So long as $P_c > P_{atm}$, the projectile will keep accelerating towards the exit.

Ideally the barrel is long enough to allow the pressure chamber to reach P_{atm} .

To find the appropriate size I will assume:

- The projectile works as a perfect seal.
- No friction projectile-barrel ($F_{f1} = \mu N = 0$).
- No friction projectile-air ($F_{f2} = c\dot{x} = 0$).
- Adiabatic expansion of the air.
- Air is an ideal gas ($\frac{C_p}{C_v} = 1.38$).
- No losses in pressure due to bends and such.
- Cows are spherical.



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Sizing the pressure chamber and barrel length

Modeling the problem

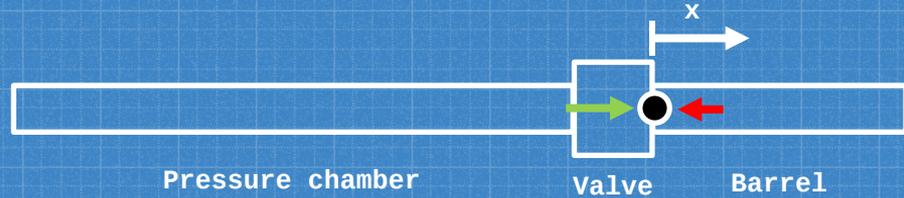
$$\sum F_x = m\ddot{x}$$

$$F_{atm} = -A_b P_{atm}$$

$$F_{exp} = +A_b P_{exp}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \rightarrow P_2 = \frac{P_1 V_1^\gamma}{V_2^\gamma} \rightarrow P_{exp} = \frac{P_i V_c^\gamma}{V_{exp}^\gamma} \{V_{exp} = V_c + A_b x\} \rightarrow P_{exp} = \frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma}$$

$$\frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma} A_b - P_{atm} A_b = m\ddot{x}$$



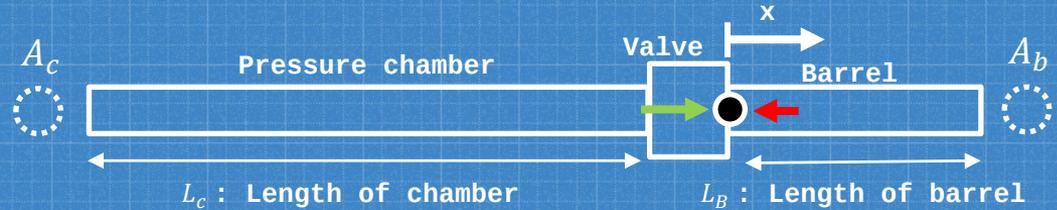
m : Mass of projectile	x : Projectile's position
\dot{x} : Projectile's speed	\ddot{x} : Projectile's acceleration
A_b : Cross section of barrel	P_{atm} : Atmospheric pressure
P_{exp} : Expanding gas's pressure	V_c : Chamber's volume
P_i : Initial chamber pressure	γ : Adiabatic expansion coefficient

NOTE: Remember to use absolute pressures for these calculations.

Sizing the pressure chamber and barrel length

Solving the differential equation

$$\ddot{x} = \frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma} A_b - P_{atm} A_b / m$$



Numerical solution

Discretization of the problem in steps with constant acceleration

$$\dot{x}_{i+1} = \ddot{x}_i \Delta t + \dot{x}_i$$

$$x_{i+1} = \frac{1}{2} \ddot{x}_i \Delta t^2 + \dot{x}_i \Delta t + x_i$$

Boundary conditions

$$\dot{x}_0 = 0 \quad \& \quad x_0 = 0$$

NOTE: Remember to use absolute pressures for these calculations.



Sizing the pressure chamber and barrel length

Results from the differential equation

$$\ddot{x} = \frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma} A_b - P_{atm} A_b}{m} \quad \{\dot{x}_0 = 0 ; x_0 = 0\}$$

$P_i = 820000 + 101325$ Pa (*relative pressure + 1 atm*). Check components rating!.

$A_b = 0,001963$ m² (*50mm diameter tubes*).

$m = 0.3$ kg.

I will solve this equation for different values of

- length of the chamber (L_c) which determines the volume of the pressurized tank ($V_c = L_c A_c$) and therefore the potential energy stored in the system.
- length of the barrel (L_b) which determines the exit point for the projectile where it will stop acquiring energy from the pressure chamber.

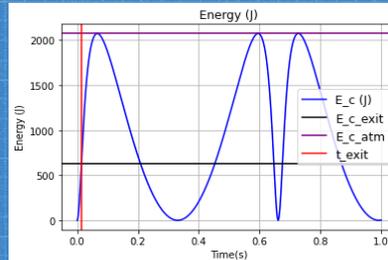
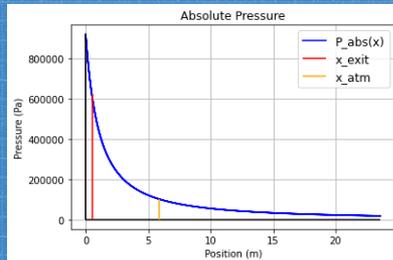
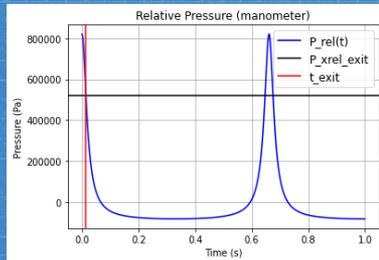
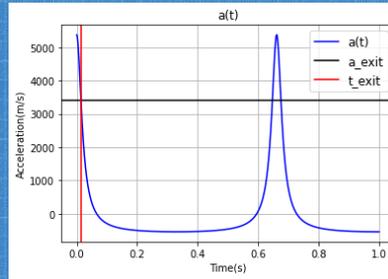
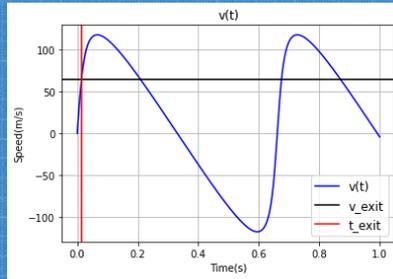
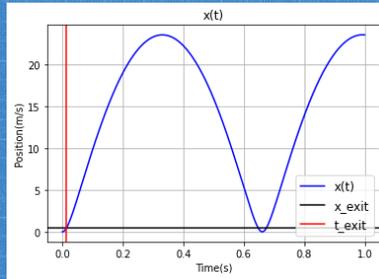


NOTE: Remember to use absolute pressures for these calculations.

Sizing the pressure chamber and barrel length

Results from the differential equation

$$L_c = 1.5\text{m} \quad \& \quad L_b = 0.5\text{m}$$



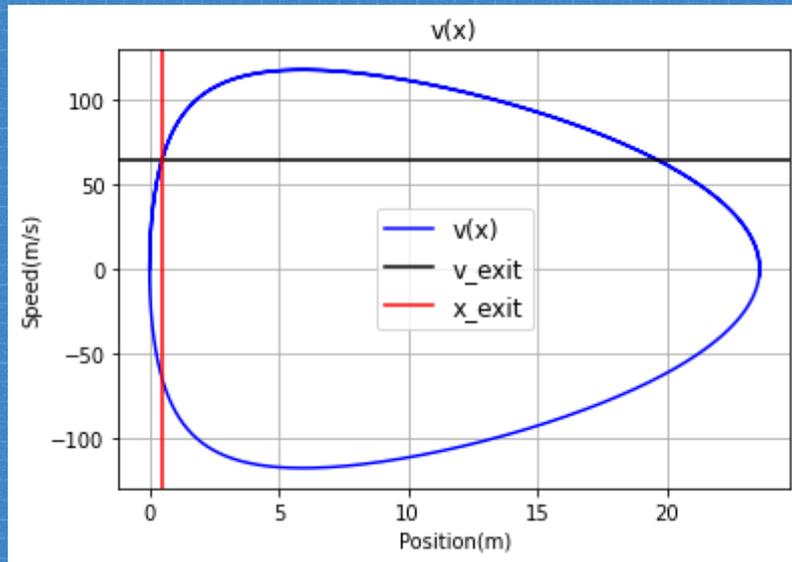
RESULTS	
Exit velocity	65 m/s
Exit energy	630 J
System's exergy	2074 J
Energy efficiency w.r.t. exergy	30%
Total energy of the compressed gas	7140 J
Energy efficiency w.r.t. energy	8%
I could increase the length of the barrel and reduce the chamber.	

NOTE: Remember to use absolute pressures for these calculations.

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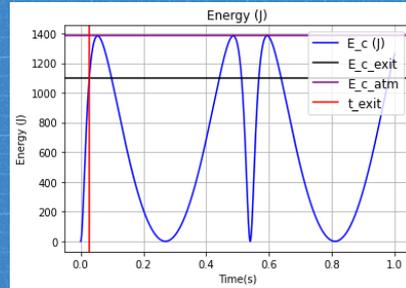
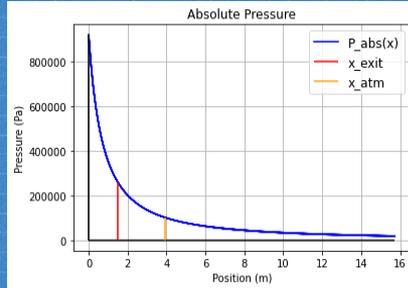
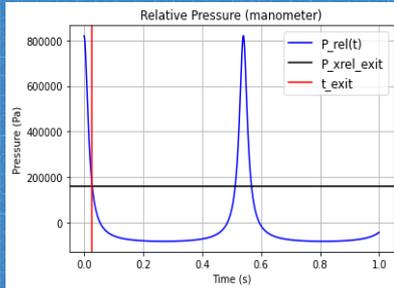
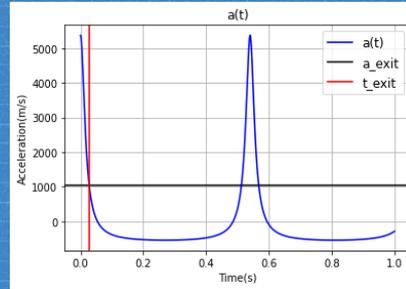
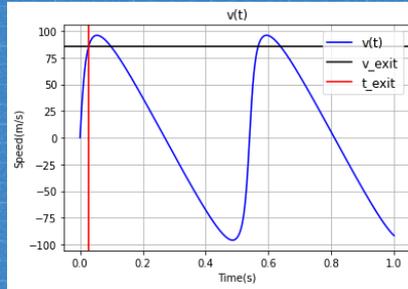
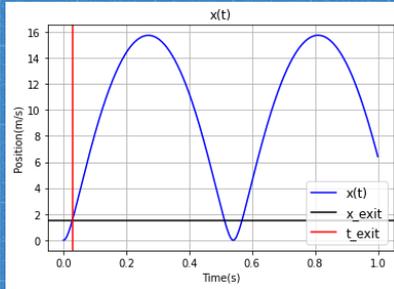
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Sizing the pressure chamber and barrel length

Results from the differential equation

$$L_c = 1\text{m} \quad \& \quad L_b = 1.5\text{m}$$



RESULTS

Exit velocity	85 m/s
Exit energy	1098 J
System's exergy	1382 J
Energy efficiency w.r.t. exergy	79%
Total energy of the compressed gas	4760 J
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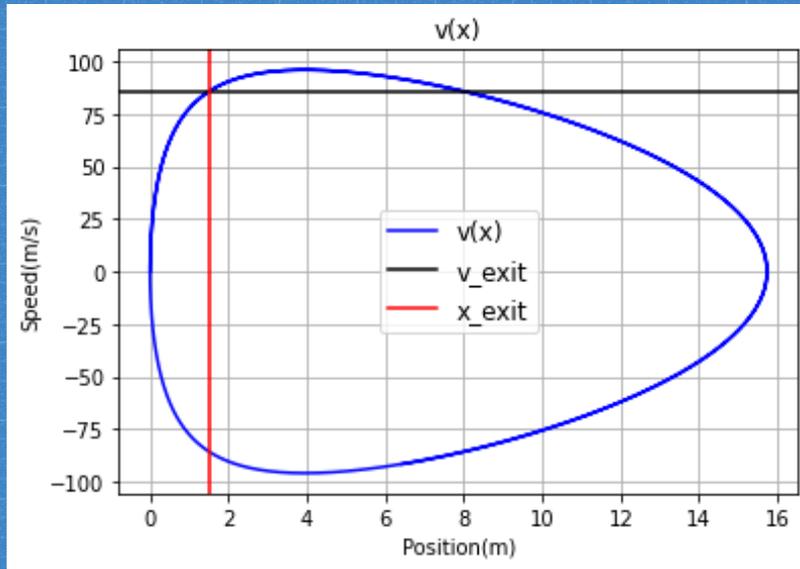
More efficient use of energy to achieve about the same exiting speed as before. Smaller pressure chamber means less energy → Less pumping and safer

NOTE: Remember to use absolute pressures for these calculations.

Sizing the pressure chamber and barrel length

Results from the differential equation

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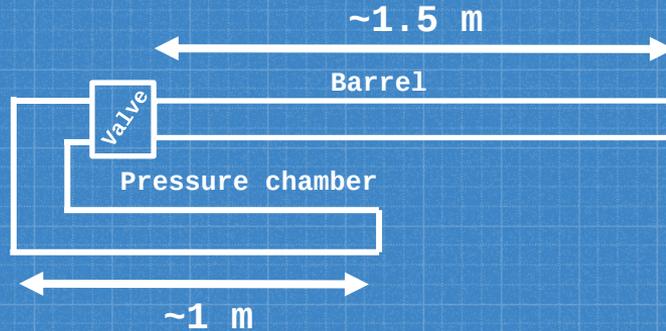


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More efficient use of energy to achieve about the same exiting speed as before. Smaller pressure chamber means less energy \rightarrow Less pumping and safer

Final concept for the pneumatic cannon



Characteristics		
MAX pressure (Check components!)	820 kPa	120 PSI
Exit velocity (m≈0.3kg)	85 m/s	279 ft/s
Exit energy	1098 J	-
System's exergy	1382 J	-
Energy efficiency w.r.t. exergy	79%	-



Related recommended videos

Channel	Video	Thumbnail	Comments
Hardware Unknown	How to Make a Simple Air Cannon		Straight to the point. Clear instructions & materials used. Modified sprinkler valve for improved flow. Emphasis on safety instructions when handling it.
Pneumatic launcher (aka "Air Cannon")	Halfmoon TechLabser (aka "Air Cannon")		Build of a U-shape cannon to have a bigger pressure chamber. It keeps the original solenoid from the sprinkler valve.
Woods2Table	DOES BARREL LENGTH MATTER for a Bait Cannon??? How to get the MAXIMUM DISTANCE for a BAIT CANNON!		Actual empirical tests to check the distance travelled by the projectile as a function of the length of the barrel for a given system (<i>pressure chamber size, diameter of the cannon, initial pressure of the system, etc.</i>). Remember: "In theory, theory and practice are the same. In practice, they are not."



Thanks!

Now let's engage in some interesting conversation in the comments section.

