

MATH  
PHYSICS  
ENGINEERING

*HOBBYIST*

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Air cannon fundamentals  
&  
Optimized barrel length

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# OBJECTIVE

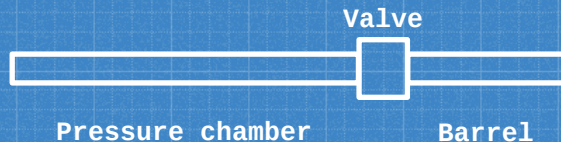
To create a hypothetical homemade cannon out of appropriately rated PVC pipes and components.

- Projectiles will be potato like in terms of mass, size and shape.
- Achieve a projectile exiting velocity between 75-100 m/s (246-328 ft/s).
- Maximum pressure will be 820 kPa (120 PSI) - Check components rating for the build!.
- Barrel length and pressure chamber size will be optimized.





# CANNON' SOURCE OF ENERGY



Combustion	
Pros	Cons
<ul style="list-style-type: none"> <li>• BOOM is cool</li> <li>• No valve is needed</li> </ul>	<ul style="list-style-type: none"> <li>• Unreliable stoichiometry</li> <li>• Availability of fuel</li> </ul>

Dry ice	
Pros	Cons
<ul style="list-style-type: none"> <li>• Dry ice is cool <small>(get it?)</small></li> </ul>	<ul style="list-style-type: none"> <li>• Availability of fuel</li> <li>• Opening time of the valve is key</li> </ul>

Pressurized air	
Pros	Cons
<ul style="list-style-type: none"> <li>• Availability of fuel</li> <li>• Reliable pressure</li> </ul>	<ul style="list-style-type: none"> <li>• Pressurizing air takes time</li> <li>• Opening time of the valve is key</li> </ul>

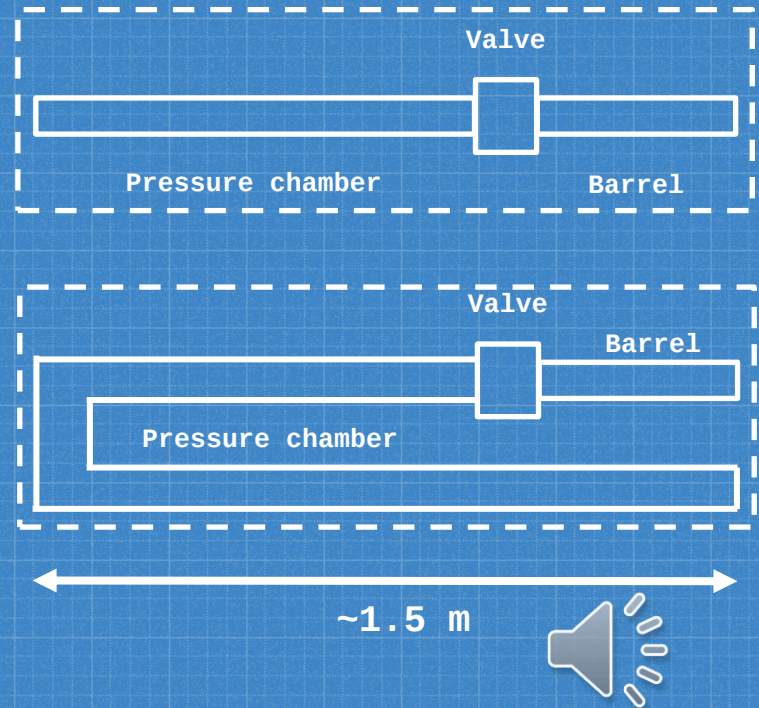
Combustion + Pressurized air	
Pros	Cons
<ul style="list-style-type: none"> <li>• BIG BOOM is cooler than BOOM</li> </ul>	<ul style="list-style-type: none"> <li>• Outright dangerous and scary</li> <li>• Stronger materials for extra safety</li> </ul>





# Overall shape for the pneumatic cannon

- Total length ~1.5 m (590 in).
- Barrel diameter 50 mm (1.96 in).
- Ergonomic to be aimed from the hip.
- Pressure chamber to barrel volume ratio is TBD.





# Sizing the pressure chamber and barrel length

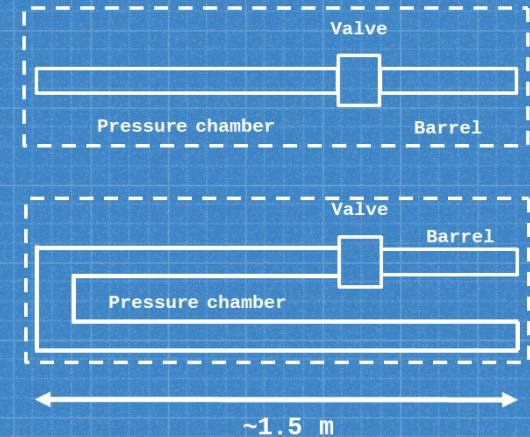
## Problem description and simplifications

So long as  $P_c > P_{atm}$ , the projectile will keep accelerating towards the exit.

Ideally the barrel is long enough to allow the pressure chamber to reach  $P_{atm}$ .

To find the appropriate size I will assume:

- The projectile works as a perfect seal.
- No friction projectile-barrel ( $F_{f1} = \mu N = 0$ ).
- No friction projectile-air ( $F_{f2} = c\dot{x} = 0$ ).
- Adiabatic expansion of the air.
- Air is an ideal gas ( $\frac{C_p}{C_v} = 1.38$ ).
- No losses in pressure due to bends and such.
- Cows are spherical.



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# Sizing the pressure chamber and barrel length

## Modeling the problem

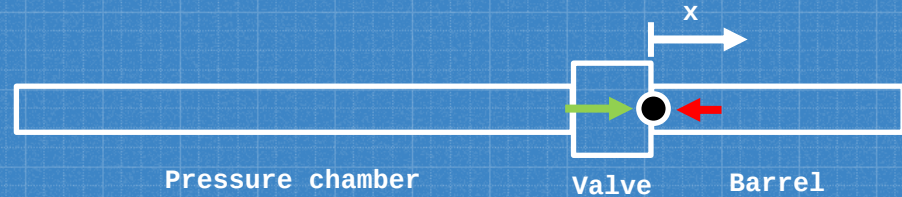
$$\sum F_x = m\ddot{x}$$

$$F_{atm} = -A_b P_{atm}$$

$$F_{exp} = +A_b P_{exp}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \rightarrow P_2 = \frac{P_1 V_1^\gamma}{V_2^\gamma} \rightarrow P_{exp} = \frac{P_i V_c^\gamma}{V_{exp}^\gamma} \{V_{exp} = V_c + A_b x\} \rightarrow P_{exp} = \frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma}$$

$$\frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma} A_b - P_{atm} A_b = m\ddot{x}$$



$m$ : Mass of projectile	$x$ : Projectile's position
$\dot{x}$ : Projectile's speed	$\ddot{x}$ : Projectile's acceleration
$A_b$ : Cross section of barrel	$P_{atm}$ : Atmospheric pressure
$P_{exp}$ : Expanding gas's pressure	$V_c$ : Chamber's volume
$P_i$ : Initial chamber pressure	$\gamma$ : Adiabatic expansion coefficient

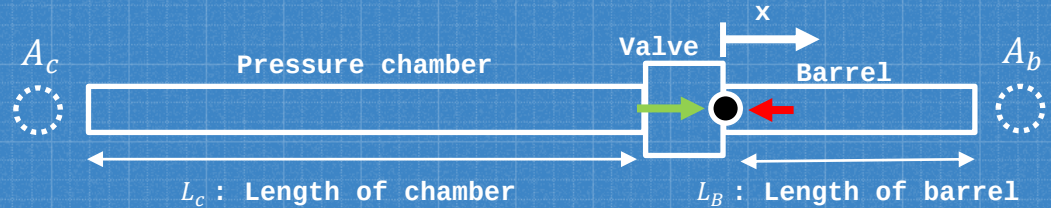
NOTE: Remember to use absolute pressures for these calculations.



# Sizing the pressure chamber and barrel length

## Solving the differential equation

$$\ddot{x} = \frac{\frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma} A_b - P_{atm} A_b}{m}$$



Numerical solution

Discretization of the problem in steps with constant acceleration

$$\dot{x}_{i+1} = \ddot{x}_i \Delta t + \dot{x}_i$$

$$x_{i+1} = \frac{1}{2} \ddot{x}_i \Delta t^2 + \dot{x}_i \Delta t + x_i$$

Boundary conditions

$$\dot{x}_0 = 0 \quad \& \quad x_0 = 0$$

*NOTE: Remember to use absolute pressures for these calculations.*





# Sizing the pressure chamber and barrel length

## Results from the differential equation

$$\ddot{x} = \frac{\frac{P_i V_c^\gamma}{(V_c + A_b x)^\gamma} A_b - P_{atm} A_b}{m} \quad \{\dot{x}_0 = 0 ; x_0 = 0\}$$

$P_i = 820000 + 101325$  Pa (*relative pressure + 1 atm*). Check components rating!.

$A_b = 0,001963$  m<sup>2</sup> (*50mm diameter tubes*).

$m = 0.3$  kg.

I will solve this equation for different values of

- length of the chamber ( $L_c$ ) which determines the volume of the pressurized tank ( $V_c = L_c A_c$ ) and therefore the potential energy stored in the system.
- length of the barrel ( $L_b$ ) which determines the exit point for the projectile where it will stop acquiring energy from the pressure chamber.



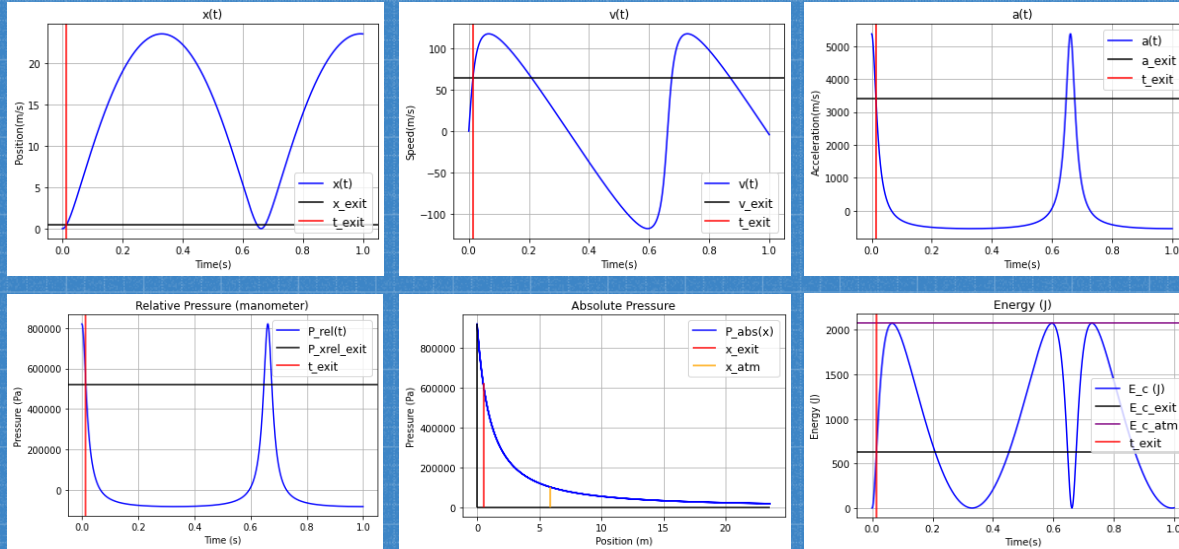
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# Sizing the pressure chamber and barrel length

## Results from the differential equation

$$L_c = 1.5\text{m} \quad \& \quad L_b = 0.5\text{m}$$



RESULTS	
Exit velocity	65 m/s
Exit energy	630 J
System's exergy	2074 J
Energy efficiency w.r.t. exergy	30%
Total energy of the compressed gas	7140 J
Energy efficiency w.r.t. energy	8%
I could increase the length of the barrel and reduce the chamber.	

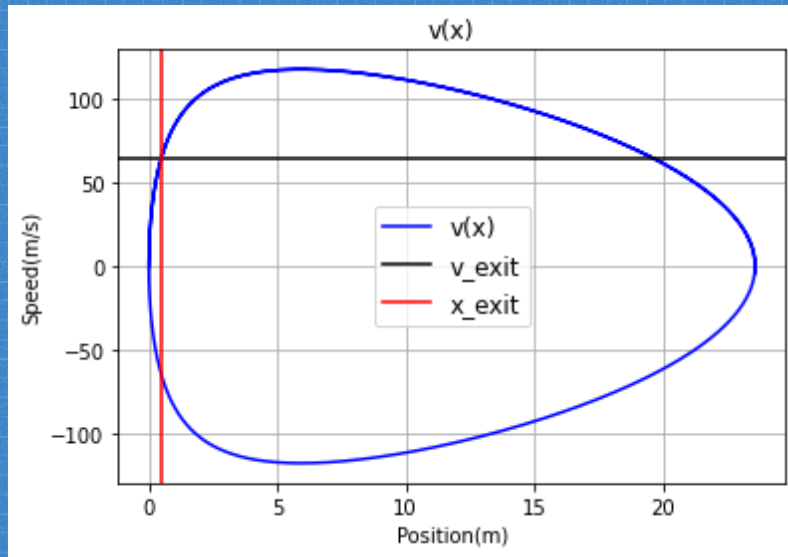
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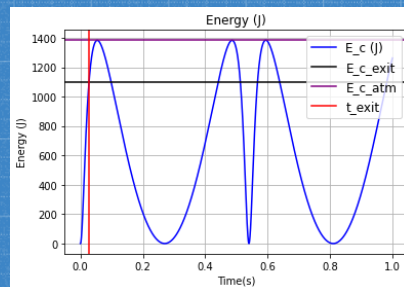
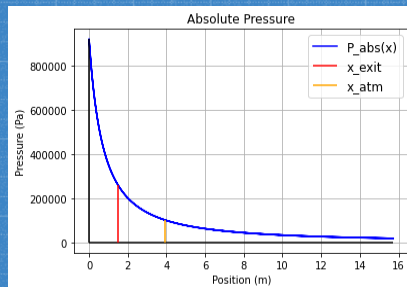
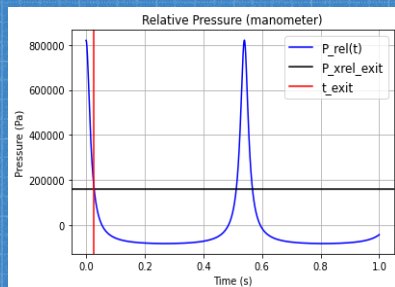
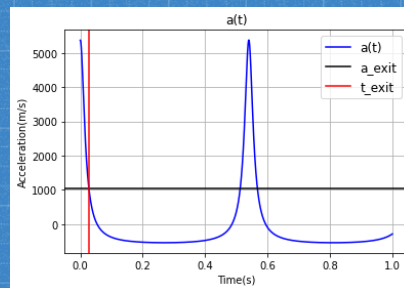
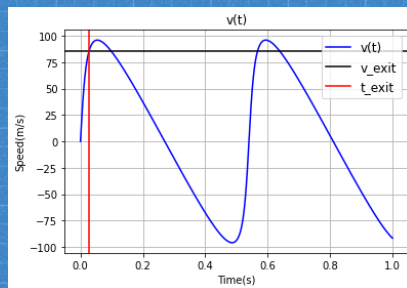
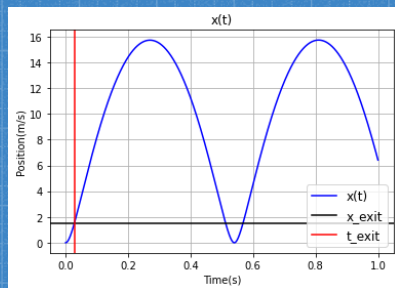
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# Sizing the pressure chamber and barrel length

## Results from the differential equation

$$L_c = 1\text{m} \quad \& \quad L_b = 1.5\text{m}$$



### RESULTS

Exit velocity	85 m/s
Exit energy	1098 J
System's exergy	1382 J
Energy efficiency w.r.t. exergy	79%
Total energy of the compressed gas	4760 J
Energy efficiency w.r.t. energy	23%

More efficient use of energy to achieve about the same exiting speed as before. Smaller pressure chamber means less energy → Less pumping and safer

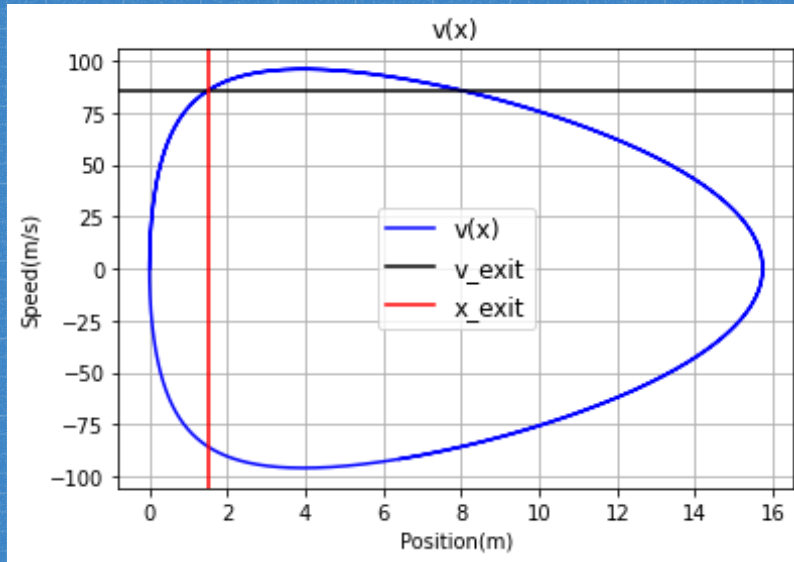
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# Sizing the pressure chamber and barrel length

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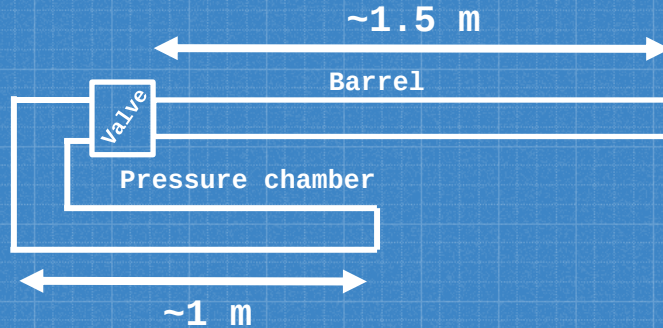


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MPE Hobbyist

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# Final concept for the pneumatic cannon



Characteristics		
MAX pressure (Check components!)	820 kPa	120 PSI
Exit velocity ( $m \approx 0.3\text{kg}$ )	85 m/s	279 ft/s
Exit energy	1098 J	-
System's exergy	1382 J	-
Energy efficiency w.r.t. exergy	79%	-





## Related recommended videos

Channel	Video	Thumbnail	Comments
Hardware Unknown	How to Make a Simple Air Cannon		Straight to the point. Clear instructions & materials used. Modified sprinkler valve for improved flow. Emphasis on safety instructions when handling it.
Pneumatic launcher (aka "Air Cannon")	Halfmoon TechLabser (aka "Air Cannon")		Build of a U-shape cannon to have a bigger pressure chamber. It keeps the original solenoid from the sprinkler valve.
Woods2Table	DOES BARREL LENGTH MATTER for a Bait Cannon??? How to get the MAXIMUM DISTANCE for a BAIT CANNON!		Actual empirical tests to check the distance travelled by the projectile as a function of the length of the barrel for a given system (pressure chamber size, diameter of the cannon, initial pressure of the system, etc.). Remember: "In theory, theory and practice are the same. In practice, they are not."





# Thanks !

Now let's engage in some interesting conversation in the comments section.

