NUI Galway
OÉ Gaillimh

Semester I Examinations 2016 / 2017

Exam Code(s)	3BM, 3BG, 3BSE
Exam(s)	3rd Mechanical Engineering 3rd Biomedical Engineering 3rd Energy Systems Engineering
Module Code	**ME301**
Module(s)	**Fluid Dynamics**
Paper No.	1
Repeat Paper	—
External Examiner(s)	Professor H. Rice
Internal Examiner(s)	Professor S.B. Leen * Dr J.A. Eaton

Instructions:

- The paper contains 5 questions. Attempt 4 questions. All questions carry equal marks.
- *For every question attempted, produce at least one sketch or illustration that is clearly and accurately labelled with symbols and appropriate dimensions. State the assumptions your analyses are based upon. Show all your workings.*
- Attached is the following information:
  - — Equations Sheet
  - — Physical Properties Tables
  - — Moody Chart & Relative Roughness Chart
  - — Standard Atmosphere Table
  - — Conical Expansion Loss
  - — Flat Plate Friction Drag

Duration	2 hours
Number of Pages	13 (including cover page)
Department(s)	Mechanical / Biomedical Engineering
Course Co-ordinator	Dr J.A. Eaton

Requirements:

Statistical/ Log Tables	Yes
Release to Library:	Yes

PTO

Question 1

(a) The basic differential equations of viscous fluid flow, together with their boundary conditions, may be non-dimensionalised to reveal several dimensionless parameters.

Identify three of these parameters. Describe their constituents and the ratios they represent, and briefly outline the conditions under which they are important or negligible.

[**5**]

(b) The thrust F of a free aircraft propeller depends upon fluid density ϱ, the rotation rate n in rev/s, the propeller diameter D, and the forward velocity V.

Viscous effects are slight and may be neglected here. Compressibility effects may also be ignored.

Tests of a 25-cm-diameter model aircraft propeller, in a sea-level wind tunnel, yield the following thrust data at a velocity of 20 m/s:

Rotation rate, rev/min	4,800	6,000	8,000
Measured thrust, Newton	6.1	19.0	47.0

(i) Make use of these data to produce a dimensionless plot of the test results.

[**10**]

(ii) Use the dimensionless data to predict the thrust, in Newton, of a similar 1.6-m-diameter prototype propeller when rotating at 3,800 rev/min and flying at 100 m/s at 4,000 m standard altitude.

[**5**]

PTO

Question 2

(a) When a circular pipe flow suddenly expands from A_1 to A_2, as shown in ***Figure 2***, low-speed, low-friction eddies appear in the corners and the flow gradually expands to A_2 downstream.

For steady incompressible flow, neglecting wall friction, and assuming that $p \approx p_1$ on the corner annular ring as shown, use the suggested control volume to show that the downstream pressure is given by

$$p_2 = p_1 + \rho V_1^2 \frac{A_1}{A_2}\left(1 - \frac{A_1}{A_2}\right)$$ [**8**]

(b) Verify that Bernoulli's equation is *not* valid for this sudden expansion, and that the actual head loss is given by

$$h_f \approx \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$ [**6**]

(c) Suppose that we want to analyse this sudden-pipe expansion flow using the full continuity and Navier-Stokes equations. What are the proper boundary conditions to handle this problem?

[**6**]

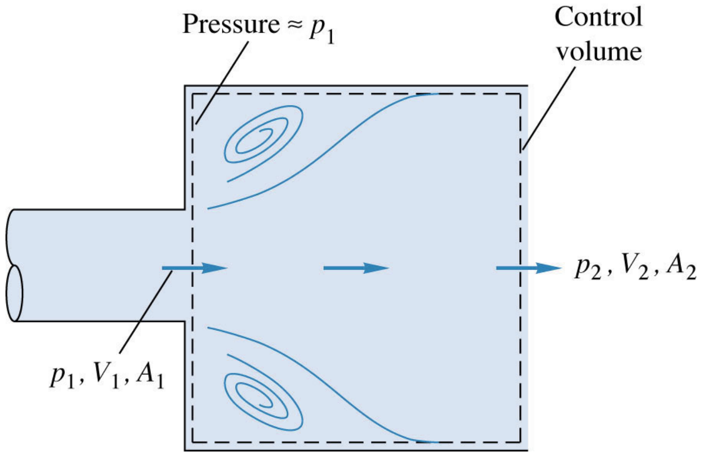


Figure 2

PTO

Question 3

(a) High-speed passenger trains are streamlined to reduce surface friction resistance. The cross-section of a passenger car of one such train is shown in ***Figure 3***.

For a train 150 m long under standard sea level conditions estimate the surface friction drag and the friction power required at speeds of

(i) 100 km/h and

(ii) 200 km/h

[10]

(b) If the same train has in addition a pressure drag coefficient for the locomotive and undercarriage of 0.8 and also a constant rolling resistance of 3,000 N, what will be the total resistance for the train at speeds of

(iii) 100 km/h and

(iv) 200 km/h ?

Assume a projected area of 9 m^2.

(c) What are the proportions of rolling resistance, pressure drag and frictional drag at these two speeds?

[10]

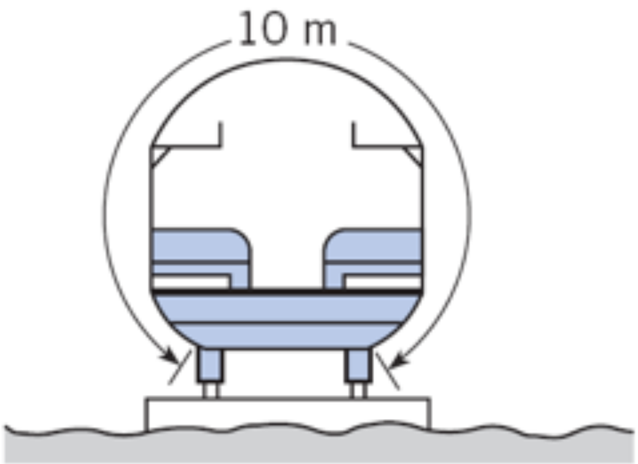


Figure 3

Question 4

A 70-percent efficient pump delivers water at 20°C from one reservoir to another 6 m higher, as in ***Figure 4***. The piping system consists of 20 m of galvanised-iron 50-mm-diameter pipe, a re-entrant entrance ($K_{entrance}$ = 1.0), two screwed 90° long-radius elbows (K_{elbow} = 0.41 each), a screwed open gate valve (K_{valve} = 0.16), and a sharp exit. The flow rate is 0.7 m^3/min.

It is desired to investigate the benefit of adding a well-designed conical expansion with a 6° total cone angle to the exit.

What is the input power required in kW :

(i) with and [**14**]

(ii) without the conical expansion added to the exit? [**6**]

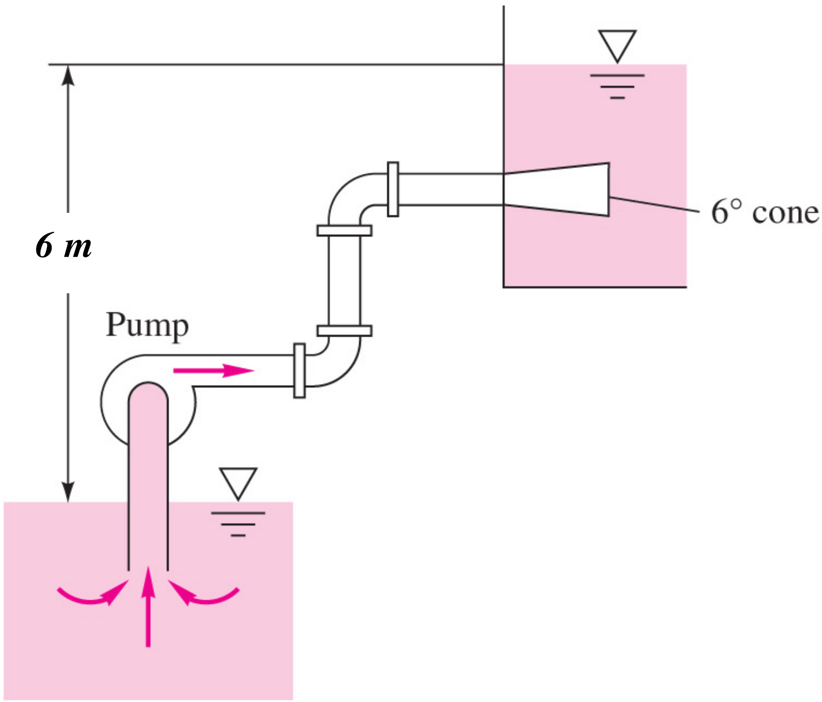


Figure 4

PTO

Question 5

(a) Derive the Rankine-Hugoniot relations for a one-dimensional compressible normal-shock wave flow. Note all assumptions used, and state the significant conclusions.

[10]

(b) For flow of air through a normal shock the upstream conditions are $V_1 = 600$ m/s, $T_{01} = 500$ K, $p_{01} = 700$ kPa.

Compute the downstream conditions:

(i) Ma_2

(ii) V_2

(iii) T_2

(iv) p_2

(v) p_{02} ,

where the symbols have their usual meanings. **[10]**

PTO

EQUATION SHEET

γ

Ideal-gas law: $p = \rho RT$, $R_{air} = 287$ J/kg-K	Surface tension: $\Delta p = Y(R_1^{-1} + R_2^{-1})$
Hydrostatics, constant density: $p_2 - p_1 = -\gamma(z_2 - z_1)$, $\gamma = \rho g$	Hydrostatic panel force: $F = \gamma h_{CG} A$, $y_{CP} = -I_{xx}\sin\theta/(h_{CG}A)$, $x_{CP} = -I_{xy}\sin\theta/(h_{CG}A)$
Buoyant force: $F_B = \gamma_{fluid}$(displaced volume)	CV mass: $d/dt(\int_{CV} \rho dv) + \sum(\rho AV)_{out} - \sum(\rho AV)_{in} = 0$
CV momentum: $d/dt(\int_{CV} \rho \mathbf{V} dv) + \sum[(\rho AV)\mathbf{V}]_{out} - \sum[(\rho AV)\mathbf{V}]_{in} = \sum \mathbf{F}$	CV angular momentum: $d/dt(\int_{CV} \rho(\mathbf{r_0} \times \mathbf{V})dv) + \sum \rho AV(\mathbf{r_0} \times \mathbf{V})_{out} - \sum \rho AV(\mathbf{r_0} \times \mathbf{V})_{in} = \sum \mathbf{M_0}$
Steady flow energy: $(p/\gamma + \alpha V^2/2g + z)_{in} = (p/\gamma + \alpha V^2/2g + z)_{out} + h_{friction} - h_{pump} + h_{turbine}$	Acceleration: $d\mathbf{V}/dt = \partial \mathbf{V}/\partial t + u(\partial \mathbf{V}/\partial x) + v(\partial \mathbf{V}/\partial y) + w(\partial \mathbf{V}/\partial z)$
Incompressible continuity: $\nabla \cdot \mathbf{V} = 0$	Navier-Stokes: $\rho(d\mathbf{V}/dt) = \rho \mathbf{g} - \nabla p + \mu \nabla^2 \mathbf{V}$
Incompressible stream function $\psi(x,y)$: $u = \partial\psi/\partial y$; $v = -\partial\psi/\partial x$	Velocity potential $\phi(x,y,z)$: $u = \partial\phi/\partial x$; $v = \partial\phi/\partial y$; $w = \partial\phi/\partial z$
Bernoulli unsteady irrotational flow: $\partial\phi/\partial t + \int dp/\rho + V^2/2 + gz = \text{Const}$	Turbulent friction factor: $1/\sqrt{f} = -2.0 \log_{10}[\varepsilon/(3.7d) + 2.51/(\mathrm{Re}_d \sqrt{f})]$
Pipe head loss: $h_f = f(L/d)V^2/(2g)$ where f = Moody chart friction factor	Orifice, nozzle, venturi flow: $Q = C_d A_{throat}[2\Delta p/\{\rho(1-\beta^4)\}]^{1/2}$, $\beta = d/D$
Laminar flat plate flow: $\delta/x = 5.0/\mathrm{Re}_x^{1/2}$, $c_f = 0.664/\mathrm{Re}_x^{1/2}$, $C_D = 1.328/\mathrm{Re}_L^{1/2}$	Turbulent flat plate flow: $\delta/x = 0.16/\mathrm{Re}_x^{1/7}$, $c_f = 0.027/\mathrm{Re}_x^{1/7}$, $C_D = 0.031/\mathrm{Re}_L^{1/7}$
$C_D = \text{Drag}/(\frac{1}{2}\rho V^2 A)$; $C_L = \text{Lift}/(\frac{1}{2}\rho V^2 A)$	2-D potential flow: $\nabla^2\phi = \nabla^2\psi = 0$
Isentropic flow: $T_0/T = 1 + \{(k-1)/2\}\mathrm{Ma}^2$, $\rho_0/\rho = (T_0/T)^{1/(k-1)}$, $p_0/p = (T_0/T)^{k/(k-1)}$	One-dimensional isentropic area change: $A/A^* = (1/Ma)[1 + \{(k-1)Ma^2/2\}/(k+1)/2]^{(1/2)(k+1)/(k-1)}$
Prandtl-Meyer expansion: $K = (k+1)/(k-1)$, $\omega = K^{1/2}\tan^{-1}[(\mathrm{Ma}^2-1)/K]^{1/2} - \tan^{-1}(\mathrm{Ma}^2-1)^{1/2}$	Uniform flow, Manning's n, SI units: $V_0(\text{m/s}) = (1.0/n)[R_h(m)]^{2/3} S_0^{1/2}$
Gradually varied channel flow: $dy/dx = (S_0 - S)/(1 - \mathrm{Fr}^2)$, $\mathrm{Fr} = V/V_{crit}$	Euler turbine formula: Power $= \rho Q(u_2 V_{t2} - u_1 V_{t1})$, $u = r\omega$

A 1/7

PTO

Table A.3 Properties of Common Liquids at 1 atm and 20°C (68°F)

Liquid	ρ, kg/m^3	μ, kg/(m · s)	Y, N/m*	p_v, N/m^2	Bulk modulus, N/m^2	Viscosity parameter $C^\dagger$
Ammonia	608	2.20 E−4	2.13 E−2	9.10 E+5	—	1.05
Benzene	881	6.51 E−4	2.88 E−2	1.01 E+4	1.4 E+9	4.34
Carbon tetrachloride	1590	9.67 E−4	2.70 E−2	1.20 E+4	9.65 E+8	4.45
Ethanol	789	1.20 E−3	2.28 E−2	5.7 E+3	9.0 E+8	5.72
Ethylene glycol	1117	2.14 E−2	4.84 E−2	1.2 E+1	—	11.7
Freon 12	1327	2.62 E−4	—	—	—	1.76
Gasoline	680	2.92 E−4	2.16 E−2	5.51 E+4	9.58 E+8	3.68
Glycerin	1260	1.49	6.33 E−2	1.4 E−2	4.34 E+9	28.0
Kerosene	804	1.92 E−3	2.8 E−2	3.11 E+3	1.6 E+9	5.56
Mercury	13,550	1.56 E−3	4.84 E−1	1.1 E−3	2.55 E+10	1.07
Methanol	791	5.98 E−4	2.25 E−2	1.34 E+4	8.3 E+8	4.63
SAE 10W oil	870	1.04 E−1‡	3.6 E−2	—	1.31 E+9	15.7
SAE 10W30 oil	876	1.7 E−1‡	—	—	—	14.0
SAE 30W oil	891	2.9 E−1‡	3.5 E−2	—	1.38 E+9	18.3
SAE 50W oil	902	8.6 E−1‡	—	—	—	20.2
Water	998	1.00 E−3	7.28 E−2	2.34 E+3	2.19 E+9	Table A.1
Seawater (30%)	1025	1.07 E−3	7.28 E−2	2.34 E+3	2.33 E+9	7.28

*In contact with air.

†The viscosity–temperature variation of these liquids may be fitted to the empirical expression

$$\frac{\mu}{\mu_{20°C}} \approx \exp\left[C\left(\frac{293\text{ K}}{T\text{ K}} - 1\right)\right]$$

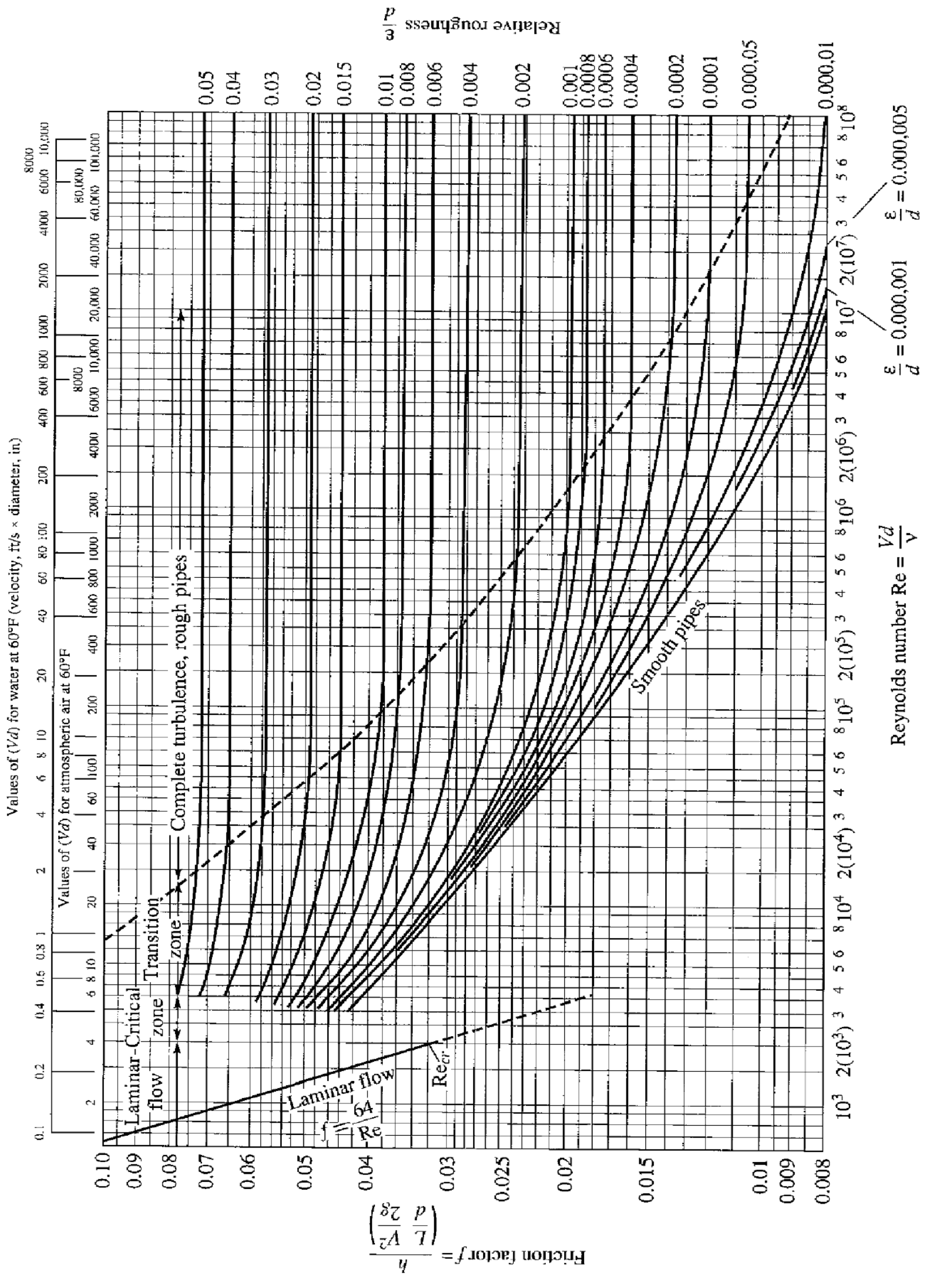
with accuracy of ±6 percent in the range $0 \le T \le 100°C$.

‡Representative values. The SAE oil classifications allow a viscosity variation of up to ±50 percent, especially at lower temperatures.

Table A.4 Properties of Common Gases at 1 atm and 20°C (68°F)

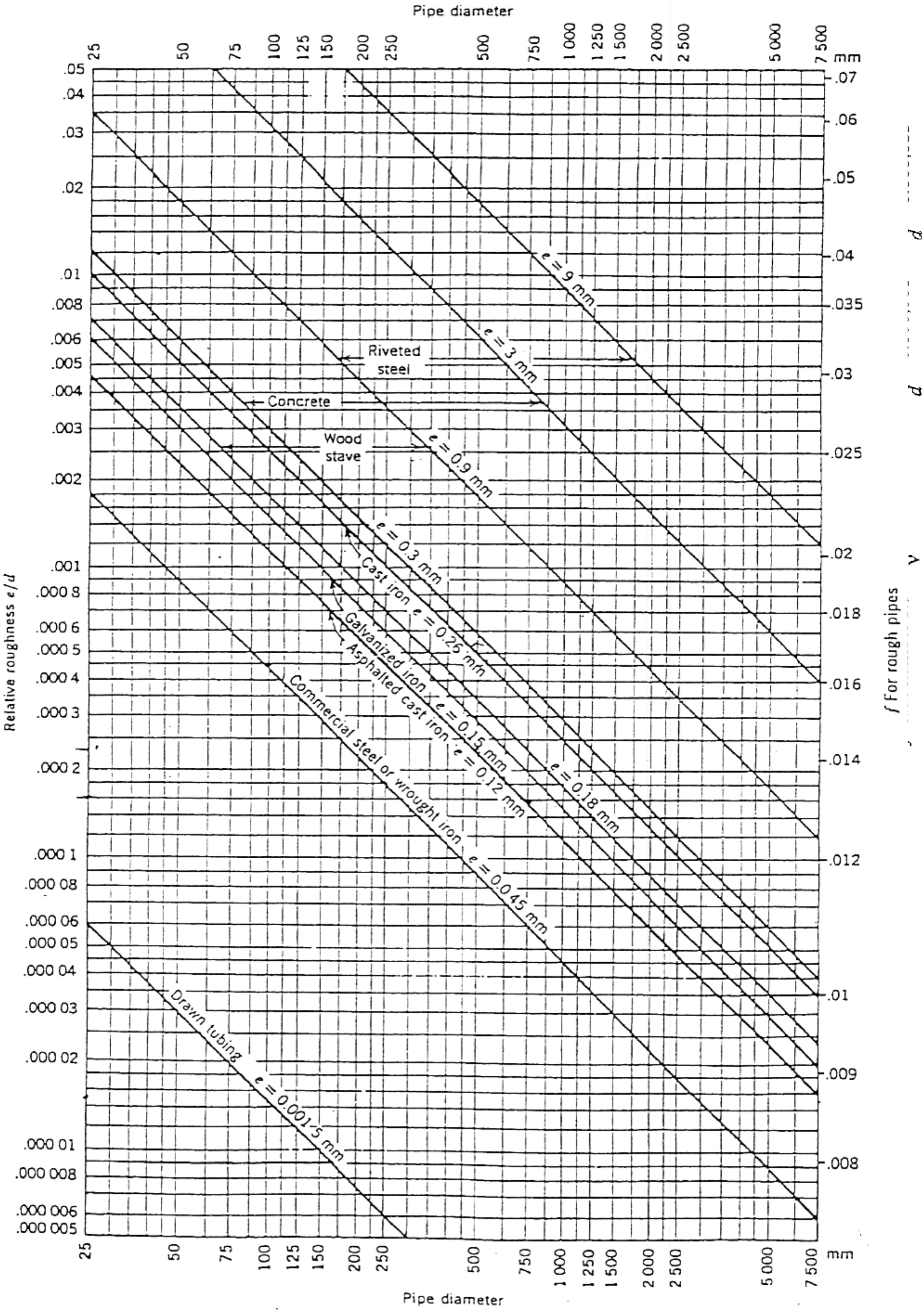
Gas	Molecular weight	R, m^2/(s^2 · K)	ρg, N/m^3	μ, N · s/m^2	Specific-heat ratio	Power-law exponent n^*
H_2	2.016	4124	0.822	9.05 E−6	1.41	0.68
He	4.003	2077	1.63	1.97 E−5	1.66	0.67
H_2O	18.02	461	7.35	1.02 E−5	1.33	1.15
Ar	39.944	208	16.3	2.24 E−5	1.67	0.72
Dry air	28.96	287	11.8	1.80 E−5	1.40	0.67
CO_2	44.01	189	17.9	1.48 E−5	1.30	0.79
CO	28.01	297	11.4	1.82 E−5	1.40	0.71
N_2	28.02	297	11.4	1.76 E−5	1.40	0.67
O_2	32.00	260	13.1	2.00 E−5	1.40	0.69
NO	30.01	277	12.1	1.90 E−5	1.40	0.78
N_2O	44.02	189	17.9	1.45 E−5	1.31	0.89
Cl_2	70.91	117	28.9	1.03 E−5	1.34	1.00
CH_4	16.04	518	6.54	1.34 E−5	1.32	0.87

*The power-law curve fit, Eq. (1.27), $\mu/\mu_{293K} \approx (T/293)^n$, fits these gases to within ±4 percent in the range $250 \le T \le 1000$ K. The temperature must be in kelvins.

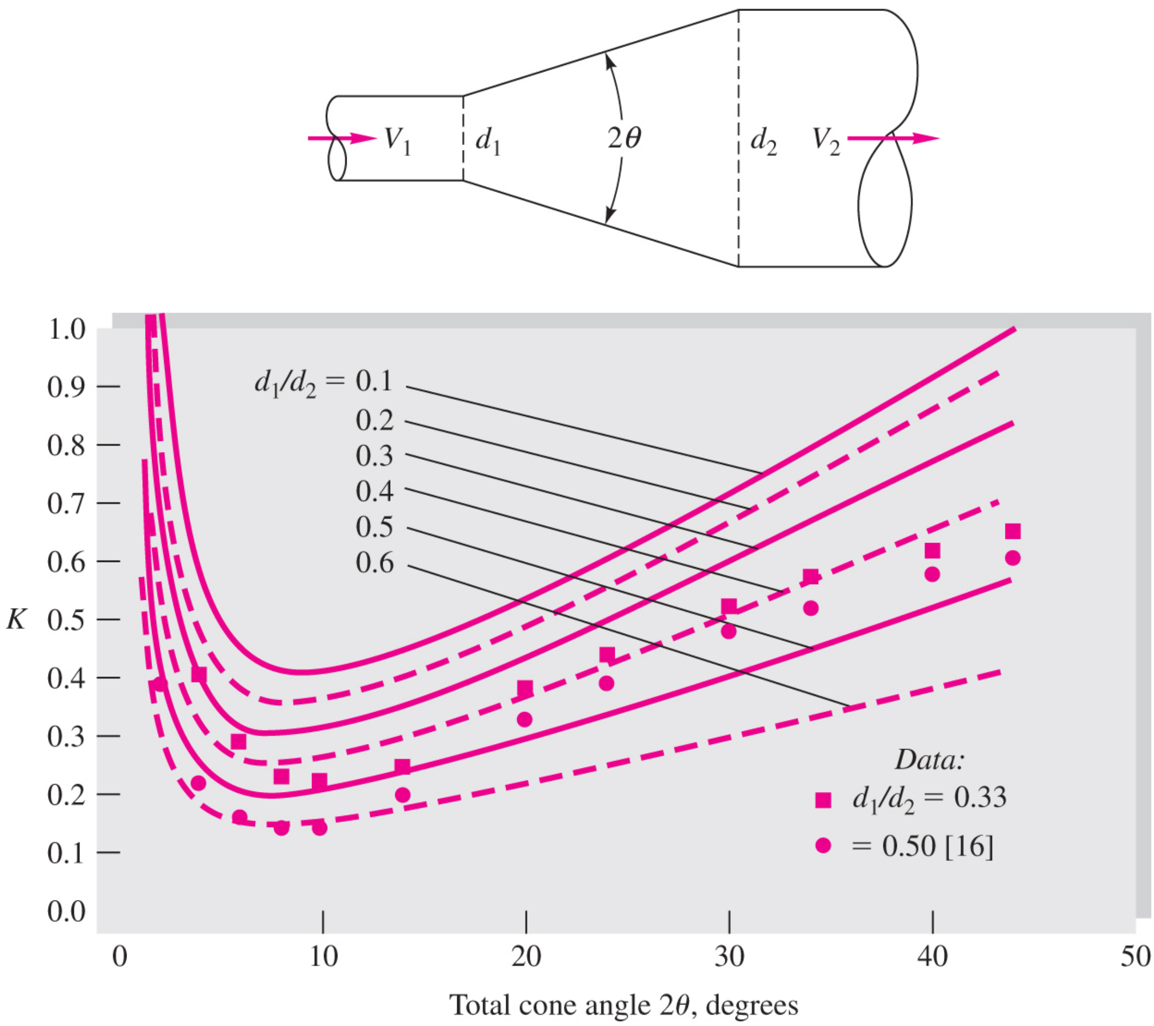


The Moody chart for pipe friction with rough and smooth walls.

PTO



Relative roughness chart



White, Figure 6.23   Flow losses in a gradual conical expansion

PTO

Table A.6 Properties of the Standard Atmosphere

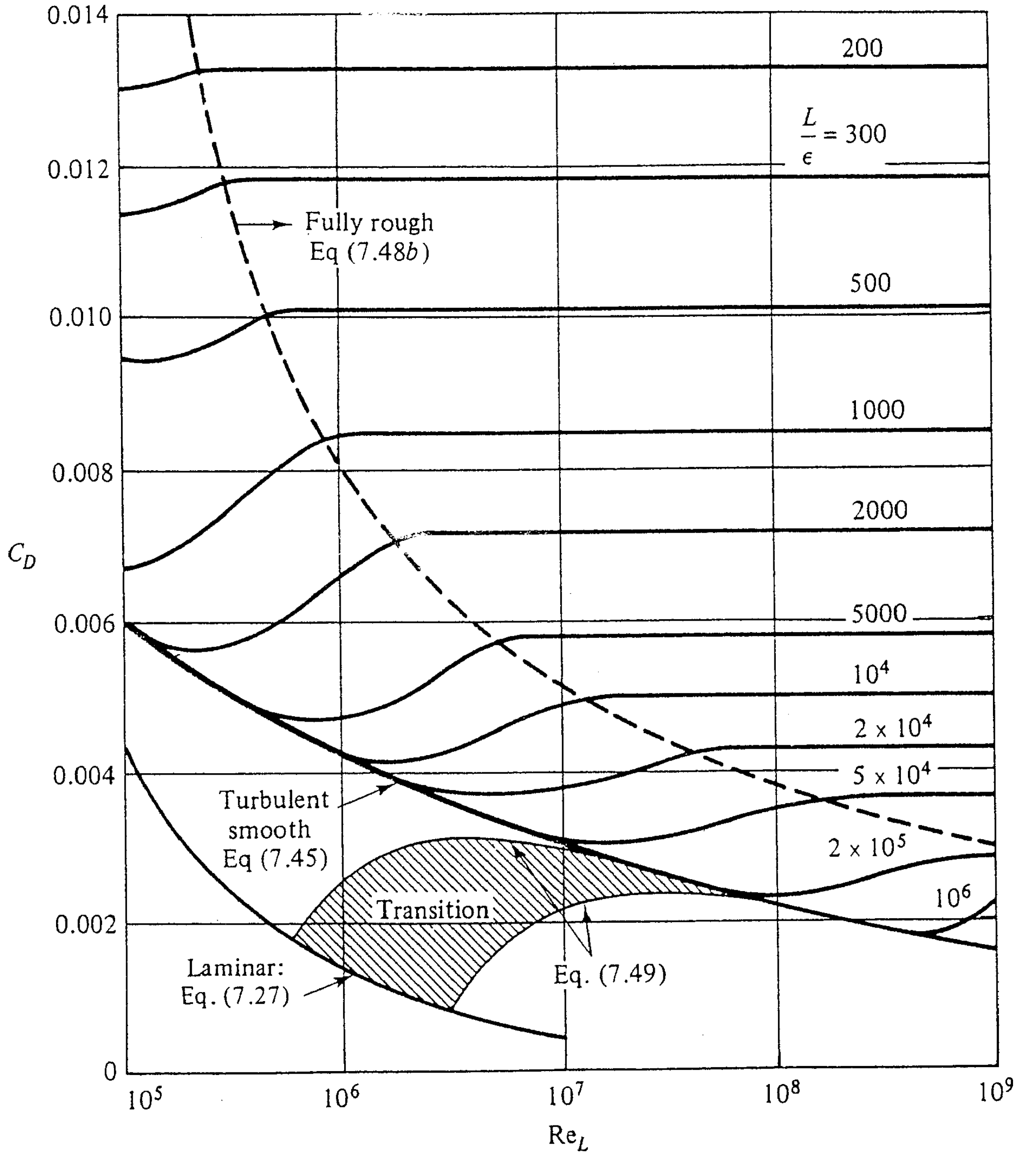
z, m	*T*, K	*p*, Pa	ρ, kg/m^3	*a*, m/s
−500	291.41	107,508	1.2854	342.2
0	288.16	101,350	1.2255	340.3
500	284.91	95,480	1.1677	338.4
1000	281.66	89,889	1.1120	336.5
1500	278.41	84,565	1.0583	334.5
2000	275.16	79,500	1.0067	332.6
2500	271.91	74,684	0.9570	330.6
3000	268.66	70,107	0.9092	328.6
3500	265.41	65,759	0.8633	326.6
4000	262.16	61,633	0.8191	324.6
4500	258.91	57,718	0.7768	322.6
5000	255.66	54,008	0.7361	320.6
5500	252.41	50,493	0.6970	318.5
6000	249.16	47,166	0.6596	316.5
6500	245.91	44,018	0.6237	314.4
7000	242.66	41,043	0.5893	312.3
7500	239.41	38,233	0.5564	310.2
8000	236.16	35,581	0.5250	308.1
8500	232.91	33,080	0.4949	306.0
9000	229.66	30,723	0.4661	303.8
9500	226.41	28,504	0.4387	301.7
10,000	223.16	26,416	0.4125	299.5
10,500	219.91	24,455	0.3875	297.3
11,000	216.66	22,612	0.3637	295.1
11,500	216.66	20,897	0.3361	295.1
12,000	216.66	19,312	0.3106	295.1
12,500	216.66	17,847	0.2870	295.1
13,000	216.66	16,494	0.2652	295.1
13,500	216.66	15,243	0.2451	295.1
14,000	216.66	14,087	0.2265	295.1
14,500	216.66	13,018	0.2094	295.1
15,000	216.66	12,031	0.1935	295.1
15,500	216.66	11,118	0.1788	295.1
16,000	216.66	10,275	0.1652	295.1
16,500	216.66	9496	0.1527	295.1
17,000	216.66	8775	0.1411	295.1
17,500	216.66	8110	0.1304	295.1
18,000	216.66	7495	0.1205	295.1
18,500	216.66	6926	0.1114	295.1
19,000	216.66	6401	0.1029	295.1
19,500	216.66	5915	0.0951	295.1
20,000	216.66	5467	0.0879	295.1
22,000	218.6	4048	0.0645	296.4
24,000	220.6	2972	0.0469	297.8
26,000	222.5	2189	0.0343	299.1
28,000	224.5	1616	0.0251	300.4
30,000	226.5	1197	0.0184	301.7
40,000	250.4	287	0.0040	317.2
50,000	270.7	80	0.0010	329.9
60,000	255.7	22	0.0003	320.6
70,000	219.7	6	0.0001	297.2

Table A.1 Viscosity and Density of Water at 1 atm

T, °C	ρ, kg/m^3	μ, N · s/m^2	ν, m^2/s	*T*, °F	ρ, slug/ft^3	μ, lb · s/ft^2	ν, ft^2/s
0	1000	1.788 E−3	1.788 E−6	32	1.940	3.73 E−5	1.925 E−5
10	1000	1.307 E−3	1.307 E−6	50	1.940	2.73 E−5	1.407 E−5
20	998	1.003 E−3	1.005 E−6	68	1.937	2.09 E−5	1.082 E−5
30	996	0.799 E−3	0.802 E−6	86	1.932	1.67 E−5	0.864 E−5
40	992	0.657 E−3	0.662 E−6	104	1.925	1.37 E−5	0.713 E−5
50	988	0.548 E−3	0.555 E−6	122	1.917	1.14 E−5	0.597 E−5
60	983	0.467 E−3	0.475 E−6	140	1.908	0.975 E−5	0.511 E−5
70	978	0.405 E−3	0.414 E−6	158	1.897	0.846 E−5	0.446 E−5
80	972	0.355 E−3	0.365 E−6	176	1.886	0.741 E−5	0.393 E−5
90	965	0.316 E−3	0.327 E−6	194	1.873	0.660 E−5	0.352 E−5
100	958	0.283 E−3	0.295 E−6	212	1.859	0.591 E−5	0.318 E−5

A 6 / 7

PTO



Drag coefficient of laminar and turbulent boundary layer on smooth and rough flat plates

Table 2 Approximate Properties of Some Common Gases

	SI Units				
	Engineering Gas Constant, R, J/kg · K	Universal Gas Constant, $\mathfrak{R} = mR$, J/kg · K	Adiabatic Exponent, k —	Specific Heat at Constant Pressure, c_p, J/kg · K	Viscosity at 20°C, $\mu \times 10^5$, Pa · s
Carbon dioxide	187.8	8 264	1.28	858.2	1.47
Oxygen	259.9	8 318	1.40	909.2	2.01
Air	286.8	8 313	1.40	1 003	1.81
Nitrogen	296.5	8 302	1.40	1 038	1.76
Methane	518.1	8 302	1.31	2 190	1.34
Helium	2 076.8	8 307	1.66	5 223	1.97
Hydrogen	4 126.6	8 318	1.40	14 446	0.90