High-Energy Behavior

To analyze the high-energy behavior of the Compton scattering cross section, it is easiest to work in the center-of-mass frame. We can easily construct the differential cross section in this frame from the invariant expression (5.87). The kinematics of the reaction now looks like this:

Plugging these values into (5.87), we see that for $\theta \approx \pi$, the term $p{\cdot}k/p{\cdot}k'$ becomes very large, while the other terms are all of $\mathcal{O}(1)$ or smaller. Thus for $E \gg m$ and $\theta \approx \pi$, we have

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 \approx 2e^4 \cdot \frac{p\cdot k}{p\cdot k'} = 2e^4 \cdot \frac{E+\omega}{E+\omega\cos\theta}. \tag{5.93}$$

The cross section in the CM frame is given by (4.84):

$$\begin{aligned}\frac{d\sigma}{d\cos\theta} &= \frac{1}{2}\cdot\frac{1}{2E}\cdot\frac{1}{2\omega}\cdot\frac{\omega}{(2\pi)4(E+\omega)}\cdot\frac{2e^4(E+\omega)}{E+\omega\cos\theta}\\ &\approx \frac{2\pi\alpha^2}{2m^2+s(1+\cos\theta)}.\end{aligned} \tag{5.94}$$

Notice that, since $s \gg m^2$, the denominator of (5.94) almost vanishes when the photon is emitted in the backward direction ($\theta \approx \pi$). In fact, the electron mass m could be neglected completely in this formula if it were not necessary to cut off this singularity. To integrate over $\cos\theta$, we can drop the electron mass term if we supply an equivalent cutoff near $\theta = \pi$. In this way, we can approximate the total Compton scattering cross section by

$$\int_{-1}^{1} d(\cos\theta)\,\frac{d\sigma}{d\cos\theta} \approx \frac{2\pi\alpha^2}{s}\int_{-1+2m^2/s}^{1} d(\cos\theta)\,\frac{1}{(1+\cos\theta)}. \tag{5.95}$$

Thus, we find that the total cross section behaves at high energy as

$$\sigma_{\text{total}} = \frac{2\pi\alpha^2}{s}\log\Big(\frac{s}{m^2}\Big). \tag{5.96}$$

The main dependence α^2/s follows from dimensional analysis. But the singularity associated with backward scattering of photons leads to an enhancement by an extra logarithm of the energy.