Lecture 13 Supp. Hmwk

① Laplacian in Cylindrical

$$\nabla^2 V = 0 \qquad V = S(s)\,\Phi(\phi)\,Z(z)$$

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\frac{\partial V}{\partial s} = \frac{dS}{ds}\Phi Z \qquad \frac{\partial^2 V}{\partial \phi^2} = SZ\frac{d^2\Phi}{d\phi^2}$$

$$\frac{\partial^2 V}{\partial z^2} = S\Phi\frac{d^2 Z}{dz^2}$$

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{dS}{ds}\Phi Z\right) + \frac{1}{s^2}\frac{d^2\Phi}{d\phi^2}SZ + \frac{d^2Z}{dz^2}S\Phi = 0$$

$$\frac{1}{s}\frac{d}{ds}\left(s\frac{dS}{ds}\Phi Z\right) + \frac{1}{s^2}\frac{d^2\Phi}{d\phi^2}SZ = -\frac{d^2Z}{dz^2}S\Phi$$

$$\frac{1}{s}\frac{1}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) + \frac{1}{\Phi}\frac{1}{s^2}\frac{d^2\Phi}{d\phi^2} = -\frac{1}{Z}\frac{d^2Z}{dz^2}$$

$$\frac{1}{s}\frac{1}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) + \frac{1}{\Phi}\frac{1}{s^2}\frac{d^2\Phi}{d\phi^2} = -k^2 \qquad \hookrightarrow \text{Constant}$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = -k^2 s^2$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) + \underbrace{-n^2}_{\text{Constant}} = -k^2 s^2$$

$$\frac{d^2Z}{dz^2}\frac{1}{Z} = k^2 \quad \text{general sol.} \quad Z = Ae^{kz} + Be^{-kz}$$

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = -n^2 \quad \text{general sol.} \quad \Phi = C\sin(n\phi) + D\cos(n\phi)$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) + k^2 s^2 - n^2 = 0$$

$$s^2\frac{d^2S}{ds^2} + (k^2 s^2 - n^2)S = 0 \Rightarrow \text{Bessel Equation}$$