M is a Riemannian manifold, $\vdash$ is a global connection on M compatible with the Riemannian metric.In terms of local coordinates $u^1, ..., u^n$ defined on a coordinate neighborhood $U \subset M$, the connection $\vdash$ is determined by Γ^k_{ij} on U,as follows. Let ∂_k denote the vector field $\frac{\partial}{\partial u^k}$ on U.Then any vector field X on U can be expressed uniquely as

$$X = \sum_{k=1} x^k \partial_k$$

where the x^k are real valued functions on U.In particular the vector field $\partial_i \vdash \partial_j$ can be expressed as

$$\partial_i \vdash \partial_j = \sum_k \Gamma^k_{ij} \partial_k$$

My 1st question is that there is an equation

$$\partial_i g_{jk} =< \partial_i \vdash \partial_j, \partial_k > + < \partial_j, \partial_i \vdash \partial_k >$$

,but how could it be? The left of the equation is a vector field but the right is a function.And there are the first Christoffel identity

$$< \partial_i \vdash \partial_j, \partial_k >= \frac{1}{2}(\partial_i g_{jk} + \partial_j g_{ik} - \partial_k g_{ij})$$

and the second Christoffel identity

$$\Gamma^l_{ij} = \sum_k \frac{1}{2}(\partial_i g_{jk} + \partial_j g_{ik} - \partial_k g_{ij})$$

for which I have the same question.How could a vector field equals a real function?

My 2nd question is why in Euclidean n-space,R^n,have

$$\Gamma^k_{ij} = 0?$$

I use the second Christoffel identity but obviously it don't equals zero.The metric is the usual Riemannian metric $dx_1^2 + dx_2^2 + ... + dx_n^2$