ENGINEERING MECHANICS
(06MMB140)

Summer 2007 2 Hours

Answer **FOUR** questions, **TWO** questions from each section

Use a **SEPARATE ANSWER BOOK** for **EACH SECTION**

Any calculators are permitted

All questions carry equal marks

A '**Useful Formulae'** sheet is attached

Take the value of the acceleration due to gravity, ***g***, as 9.81m/s^2

SECTION A: MECHANICS OF MATERIALS

1. A vertical column has the horizontal cross-sectional area shown in **Figure Q 1** (dimensions in millimetres). The column carries a vertical compressive point-load of 30 kN applied at D. Calculate the following:

   (a) The second central moments of area I_{zz} and I_{xx}; [10 marks]

   (b) The maximum values of the compressive and tensile stresses; [8 marks]

   (c) The position on the cross section where the stress is zero. [7 marks]

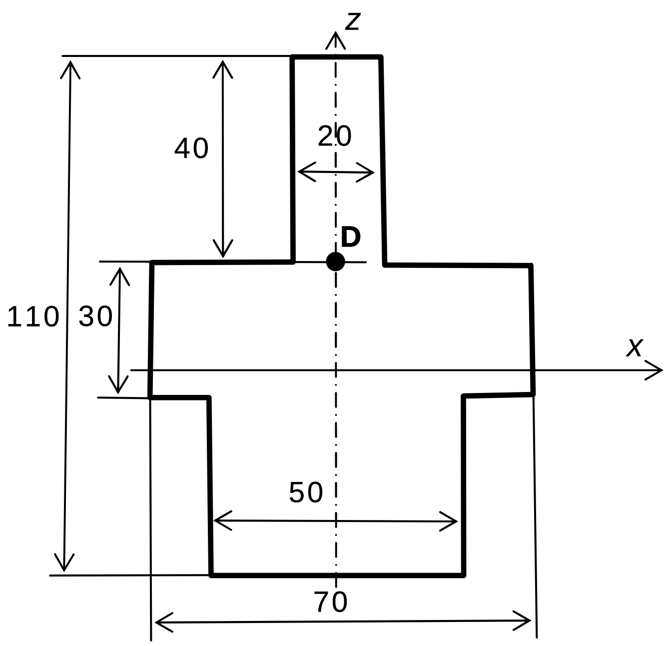

Figure Q 1 (Dimensions in millimetres)

/continued…

2. (a) A solid circular-section shaft made of Inconel 718 superalloy has a radius of 32 mm. A torque of 4 kN m is applied to the shaft. Determine the maximum shear stress and the angle of twist over a length of 70 cm. The shear modulus of Inconel 718 is 70 GN/m^2. [9 marks]

(b) The solid shaft in part (a) is to be replaced by a hollow shaft of external radius 45 mm. The hollow shaft is made of the same material. The same values of torque and maximum shear stress apply as in part (a).

(i) Determine the wall thickness of the hollow shaft. [8 marks]

(ii) Compare the weights and angles of twist of the two shafts. What are the advantages of using the hollow shaft compared with the solid shaft? [8 marks]

3. A simply supported beam is loaded as shown in **Figure Q 3**.

(a) Calculate the reaction forces at B and D. [4 marks]

(b) Draw the shear force and bending moment diagrams, showing clearly the principal values of each diagram. [14 marks]

(c) Calculate the position and value of the maximum bending moment. [7 marks]

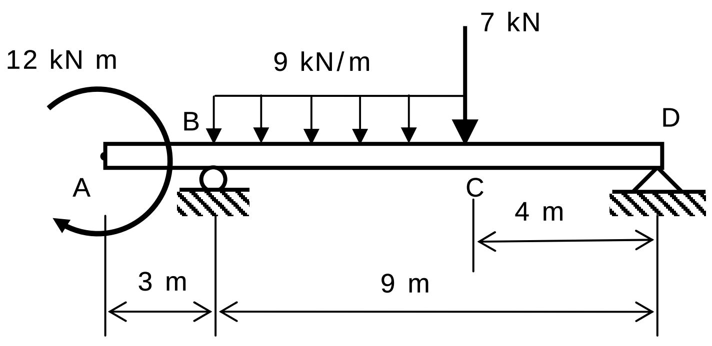

Figure Q 3

/continued...

SECTION B: DYNAMICS

4. The two identical steel balls A and B moving with initial velocities $v_A = 6m/s$ and $v_B = 8m/s$ collide, as shown in figure Q4. If the coefficient of restitution is $e = 0.7$, determine the magnitude and direction of the velocity of each ball just after impact.

[25 marks]

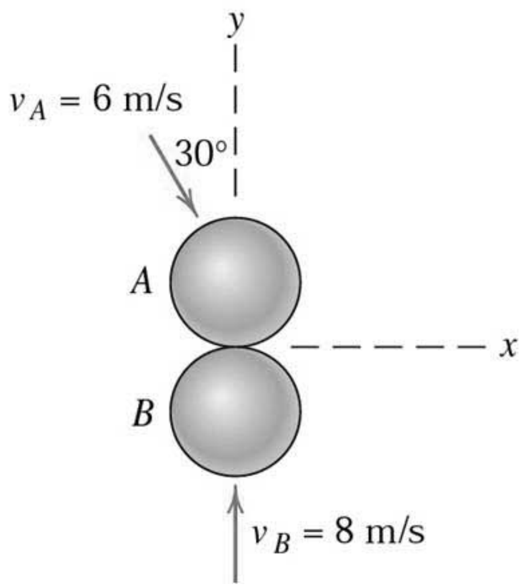

Figure Q4

/continued…

5. The 1kg cart starts rolling without friction from rest at position A and subsequently strikes the 3kg block B, as shown in Figure Q5. Block B is connected to point O with a light rigid rod, which can rotate around O. If the displacement s is 2.5 m and the coefficient of restitution for the collision is $e = 0.7$, determine the maximum angle θ that block will reach after collision. Neglect friction.

[25 marks]

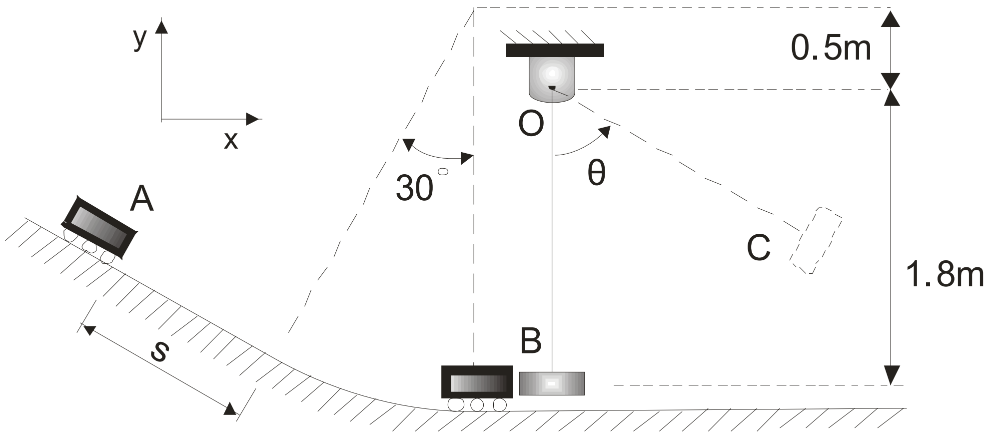

Figure Q5

/continued…

6. Determine the minimum velocity v, which the wheel of total mass m must have to just roll over the obstruction, as shown in Figure Q6. The centroidal radius of gyration of the wheel is k and it is assumed that the wheel does not slip.

[25 marks]

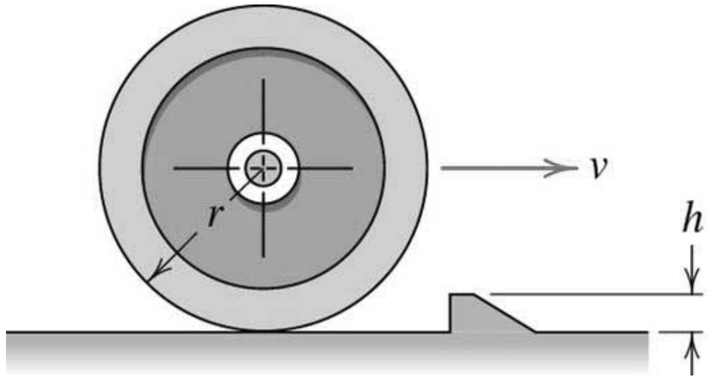

Figure Q6

A Mitrofanov
S Theodossiades

ENGINEERING MECHANICS (MMB140)
USEFUL FORMULAE

Second moments of area

$$I_{zz} = I_{z'z'} + d_y^2 A$$
$$J = I_{x'x'} + I_{y'y'}$$

Circle: $I = \dfrac{\pi d^4}{64}$

Circle: $J = \dfrac{\pi d^4}{32}$

Rectangle: $I = \dfrac{bd^3}{12}$

Friction

$$F = \mu N$$

Thermal strain

$$\varepsilon = \alpha \Delta T$$

Bending of beams

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Torsion of circular shafts

$$\frac{T}{J} = \frac{\tau}{R} = \frac{G\phi}{L}$$

PARTICLES

Rectilinear motion

Constant velocity, v : $a = 0$ and $s = vt$

Constant acceleration, a: $v = at + v_0$

and $s = \dfrac{1}{2}at^2 + v_0 t + s_0$

Plane Curvilinear Motion

Rectangular coordinates:

$$\vec{r} = x\vec{i} + y\vec{j},\ \vec{v} = \dot{x}\vec{i} + \dot{y}\vec{j},\ \vec{a} = \ddot{x}\vec{i} + \ddot{y}\vec{j}$$

Normal and tangential coordinates:

$$\vec{v} = |v|\vec{e}_t = \rho\frac{d\theta}{dt}\vec{e}_t\,, \quad \vec{a} = \frac{|v^2|}{\rho}\vec{e}_n + |\dot{v}|\vec{e}_t$$

Polar coordinates:

$$\vec{r} = r\vec{e}_r\,,\ \vec{v} = \dot{r}\,\vec{e}_r + r\dot{\theta}\,\vec{e}_\theta\,,$$
$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\vec{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\vec{e}_\theta$$

Circular motion

Normal-tangential: $v = r\dot{\theta},\ a_n = \dfrac{v^2}{r},\ a_t = r\ddot{\theta}$

Polar: $v_r = 0,\ v_\theta = r\dot{\theta},\ a_r = -r\dot{\theta}^2,\ a_\theta = r\ddot{\theta}$

Relative Motion

$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}\,,\quad \vec{v}_A = \vec{v}_B + \vec{v}_{A/B}\,,$$
$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

Newton's 2nd law

$$\Sigma\vec{F} = m\vec{a}\,,\quad \begin{matrix}\Sigma F_x = ma_x \\ \Sigma F_y = ma_y\end{matrix},\quad \begin{matrix}\Sigma F_n = ma_n \\ \Sigma F_t = ma_t\end{matrix},$$
$$\Sigma F_r = ma_r$$
$$\Sigma F_\theta = ma_\theta$$

Work / Power

$U = F \cdot d \qquad P = F \cdot v$

Work – Energy equation

$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

Linear momentum

$$\vec{G} = m \cdot \vec{v}\,,\qquad \Sigma\vec{F} = m\vec{a} = m\dot{\vec{v}} = \frac{d}{dt}(m\vec{v}) = \dot{\vec{G}}$$

Linear impulse

$$\int_{t_1}^{t_2} \Sigma\vec{F}dt = \vec{G}_2 - \vec{G}_1 = \Delta\vec{G}$$

Conservation of linear momentum

$$\Delta\vec{G} = 0 \Rightarrow \vec{G}_1 = \vec{G}_2$$

Angular momentum

$$\vec{H}_O = \vec{r} \times m\vec{v}\,,$$
$$\Sigma\vec{M}_O = \vec{r} \times \Sigma\vec{F} = \vec{r} \times m\dot{\vec{v}} = \dot{\vec{H}}_O$$

Angular impulse

$$\int_{t_1}^{t_2} \Sigma\vec{M}_O dt = \vec{H}_{O2} - \vec{H}_{O1} = \Delta\vec{H}_O$$

Conservation of angular momentum

$$\Delta\vec{H}_O = 0 \Rightarrow \vec{H}_{O1} = \vec{H}_{O2}$$

Coefficient of restitution

$$e = \frac{\hat{F}_r}{\hat{F}_d} = -\frac{v_2' - v_1'}{v_2 - v_1}$$

continued …

RIGID BODIES

Relative Motion

$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$, $v_{A/B} = r\omega$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B} = \vec{a}_B + \left(\vec{a}_{A/B}\right)_n + \left(\vec{a}_{A/B}\right)_t$

$\left(a_{A/B}\right)_n = r\omega^2$, $\left(a_{A/B}\right)_t = r\alpha$

Plane Motion

$\Sigma\vec{F} = m\vec{a}_G$, $\Sigma M_G = I_G\alpha$

$\Sigma M_P = I_G\alpha + ma_G d$

Rotation about fixed axis

$\Sigma\vec{F} = m\vec{a}_G$, $\Sigma M_G = I_G\alpha$, $\Sigma M_O = I_O\alpha$

Kinetic energy

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

Work – Energy equation

$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$

Work $U = F \cdot d$

Power $P = F \cdot v + M \cdot \omega$

Linear momentum

$\vec{G} = m \cdot \vec{v}_G$, $\Sigma\vec{F} = \dot{\vec{G}}$, $\int_{t_1}^{t_2} \Sigma\vec{F}dt = \vec{G}_2 - \vec{G}_1$

Angular momentum

$H_G = I_G\omega$, $\Sigma M_G = \dot{H}_G$,

$\int_{t_1}^{t_2} \Sigma M_G dt = H_{G2} - H_{G1}$

$H_O = I_G\omega + mv_G d$

Rotation about fixed axis

$H_O = I_O\omega$, $\Sigma M_O = I_O\dot{\omega}$,

$\int_{t_1}^{t_2} \Sigma M_O dt = I_O(\omega_2 - \omega_1)$

Conservation of momentum

$\Delta\vec{G} = 0$, $\Delta\vec{H}_O = 0$, $\Delta\vec{H}_G = 0$

Moment of inertia

$I = mk^2$, k is the radius of gyration

Uniform rod, $I_G = \dfrac{mL^2}{12}$