

In practice it is more convenient to present  $y_2$  and  $y_3$  thus:

$$y_2 = -2\sqrt{\frac{-p}{3}} \cos\left(60^\circ - \frac{\phi}{3}\right),$$

$$y_3 = -2\sqrt{\frac{-p}{3}} \cos\left(60^\circ + \frac{\phi}{3}\right).$$

**Example 1.** For the equation

$$y^3 - 3y + 1 = 0$$

we have

$$\sqrt{\frac{-p}{3}} = 1, \quad \cos \phi = -\frac{1}{2};$$

hence,  $\phi = 120^\circ$  and

$$y_1 = 2 \cos 40^\circ, \quad y_2 = -2 \cos 20^\circ, \quad y_3 = 2 \cos 80^\circ.$$

Approximate values of the roots can be found directly from the trigonometric tables and are

$$\begin{aligned} y_1 &= 1.5320888862, \\ y_2 &= -1.8793852416, \\ y_3 &= 0.3472963554. \end{aligned}$$

**Example 2.** To solve the equation

$$y^3 - 7y - 7 = 0.$$

For this equation

$$p = -7, \quad q = -7, \quad -\Delta = 49$$

and

$$\tan \phi = \frac{1}{\sqrt{27}}.$$

Further computation was made with six-place tables of logarithms. The calculations and results can be presented as follows:

$\log 27 = 1.431364$	$\log 7 = 0.845098$	$\log \cos \frac{\phi}{3} = 9.999128 - 10$
$\log \sqrt{27} = 0.715682$	$\log 3 = 0.477121$	$\frac{0.485018}{\log y_1 = 0.484146}$
$\log \tan \phi = 9.284318$	$\log \frac{7}{3} = 0.367977$	$y_1 = 3.04892$
$\phi = 10^\circ 53' 36''.2$	$\log \sqrt{\frac{7}{3}} = 0.183988$	
$\frac{1}{3}\phi = 3^\circ 37' 52''.0$	$\log 2 = 0.301030$	$\log \cos \left(60^\circ + \frac{\phi}{3}\right) = 9.647528 - 10$
	$\log 2\sqrt{\frac{7}{3}} = 0.485018$	$\frac{0.485018}{\log (-y_2) = 0.132546}$
		$-y_2 = 1.35689$
$y_1 = 3.04892$		$\log \cos \left(60^\circ - \frac{\phi}{3}\right) = 9.743387 - 10$
$y_2 = -1.35689$		$\frac{0.485018}{\log (-y_3) = 0.228405}$
$y_3 = -1.69202$		$-y_3 = 1.69202$
<u>0.00001</u>		