**none is rational, it is absolutely impossible to express any of these roots in a form involving only real radicals of any kind.**

**5. Trigonometric Solution.** In spite of the algebraic difficulties inherent in the irreducible case, it is possible to present the roots in a form suitable for numerical computation by extracting the cube root of

$$A = -\frac{q}{2} + i\sqrt{-\frac{q^2}{4} - \frac{p^3}{27}}$$

trigonometrically. The square of the modulus of $A$ is

$$\rho^2 = \left(-\frac{q}{2}\right)^2 - \left(\frac{q}{2}\right)^2 - \frac{p^3}{27} = \left(\frac{-p}{3}\right)^3$$

whence

$$\rho = \left(\frac{-p}{3}\right)^{3/2} = \frac{-p\sqrt{-p}}{\sqrt{27}}.$$

The argument of $A$ can be determined either by its cosine

$$\cos\phi = \frac{\sqrt{27}q}{2p\sqrt{-p}}$$

or by its tangent

$$\tan\phi = \frac{-\sqrt{-\Delta}}{q\sqrt{27}}$$

on the condition that $\phi$ is taken in the first or second quadrant according as $q$ is negative or positive. Having found $\rho$ and $\phi$, we can take

$$\sqrt[3]{A} = \sqrt[3]{\rho}\left(\cos\frac{\phi}{3} + i\sin\frac{\phi}{3}\right) = \sqrt{\frac{-p}{3}}\left(\cos\frac{\phi}{3} + i\sin\frac{\phi}{3}\right),$$

and

$$\sqrt[3]{B} = \sqrt{\frac{-p}{3}}\left(\cos\frac{\phi}{3} - i\sin\frac{\phi}{3}\right).$$

Then, since

$$\omega = \cos 120° + i\sin 120°,$$

the roots $y_1$, $y_2$, $y_3$ will be given by

$$y_1 = 2\sqrt{\frac{-p}{3}}\cos\frac{\phi}{3},$$

$$y_2 = 2\sqrt{\frac{-p}{3}}\cos\left(\frac{\phi}{3} + 120°\right),$$

$$y_3 = 2\sqrt{\frac{-p}{3}}\cos\left(\frac{\phi}{3} + 240°\right).$$