



## SAMPLE PROBLEM 16.1

An aluminum column of length  $L$  and rectangular cross section has a fixed end  $B$  and supports a centric load at  $A$ . Two smooth and rounded fixed plates restrain end  $A$  from moving in one of the vertical planes of symmetry of the column but allow it to move in the other plane. (a) Determine the ratio  $a/b$  of the two sides of the cross section corresponding to the most efficient design against buckling. (b) Design the most efficient cross section for the column knowing that  $L = 20$  in.,  $E = 10.1 \times 10^6$  psi,  $P = 5$  kips, and that a factor of safety of 2.5 is required.

## SOLUTION

**Buckling in  $xy$  Plane.** Referring to Fig. 16.17, we note that the effective length of the column with respect to buckling in this plane is  $L_e = 0.7L$ . The radius of gyration  $r_z$  of the cross section is obtained by writing

$$I_x = \frac{1}{12}ba^3 \quad A = ab$$

and, since  $I_z = Ar_z^2$ ,

$$r_z^2 = \frac{I_z}{A} = \frac{\frac{1}{12}ba^3}{ab} = \frac{a^2}{12} \quad r_z = a/\sqrt{12}$$

The effective slenderness ratio of the column with respect to buckling in the  $xy$  plane is

$$\frac{L_e}{r_z} = \frac{0.7L}{a/\sqrt{12}} \quad (1)$$

**Buckling in  $xz$  Plane.** The effective length of the column with respect to buckling in this plane is  $L_e = 2L$ , and the corresponding radius of gyration is  $r_y = b/\sqrt{12}$ . Thus,

$$\frac{L_e}{r_y} = \frac{2L}{b/\sqrt{12}} \quad (2)$$

**a. Most Efficient Design.** The most efficient design is that for which the critical stresses corresponding to the two possible modes of buckling are equal. Referring to Eq. (16.13'), we note that this will be the case if the two values obtained above for the effective slenderness ratio are equal. We write

$$\frac{0.7L}{a/\sqrt{12}} = \frac{2L}{b/\sqrt{12}}$$

and, solving for the ratio  $a/b$ ,

$$\frac{a}{b} = \frac{0.7}{2} \quad \frac{a}{b} = 0.35 \quad \blacktriangleleft$$

**b. Design for Given Data.** Since  $F.S. = 2.5$  is required,

$$P_{cr} = (F.S.)P = (2.5)(5 \text{ kips}) = 12.5 \text{ kips}$$

Using  $a = 0.35b$ , we have  $A = ab = 0.35b^2$  and

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12,500 \text{ lb}}{0.35b^2}$$

Making  $L = 20$  in. in Eq. (2), we have  $L_e/r_y = 138.6/b$ . Substituting for  $E$ ,  $L_e/r$ , and  $\sigma_{cr}$  into Eq. (16.13'), we write

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2} \quad \frac{12,500 \text{ lb}}{0.35b^2} = \frac{\pi^2 (10.1 \times 10^6 \text{ psi})}{(138.6/b)^2}$$

$$b = 1.620 \text{ in.} \quad a = 0.35b = 0.567 \text{ in.} \quad \blacktriangleleft$$