Undergraduate Project Report

by Owen Booler A359259

Theoretical Investigation into the Proximity effects of High Temperature superconductors

Supervised by Professor A.S.Alexandrov

Loughborough University

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1 Abstract

The proximity effect of high T_c superconductors is a new phenomenon that has deeply confused physicists. The confusion arises from the fact that conventional theories, do not seem to apply when a superconducting material comes into contact with a normal conducting material, this phenomena is called the Giant proximity effect. This report, is a theoretical investigation into the proximity effects of high temperature superconductors. Modelling the supercurrent density as a macroscopic condensate wavefunction, using an adaptation of the Gross Pitaevskii equation and the ideas of the Abrikosov vortex. Three equations for the macroscopic condensate wavefunction were derived, however due to the complexity of the equations, they cannot yet be solved. The works of London, Ginzburg and Landau were used, along with the Meissner-Ochsenfeld effect, Bose-Einstein Condensation and also the Abrikosov vortex, as a basis for the understanding of the behavior at the surface of both low and high temperature superconductors.

2 Introduction

2.1 Brief History of High T_c superconductors

When a quantum system of interacting particles behaves as though some of the particles are condensed, into a single current carrying state,it is known as a superfluid which is described by an effective wave function $\Psi(r,t)$. A superconductor is a charged superfluid.

Below a critical temperature T_c, the resistance of a superconductor falls to zero. For different elements, compounds and alloys, the critical temperature varies. These materials behave in this way because the spectrum of particles changes so that the current does not decay. There was no theory or explaination of the superconductivity phenomenon untill 1957 by the works of Bardeen, Cooper and Schrieffer. At this point frictionless flow of He^4 had already been discovered below a temperature of 2.17K, this was also known as Superfluidity.

At temperature below the critical temperature many metals, alloys, and doped semiconducting inorganic and organic compounds carry an electric current for an infinite time without any electric field.

In 1950, Ginzburg and Landau anticipated the phenomenological theory of the superconducting phase transition, this lead to the comprehension of the electromagnetic properties below T_c.

In 1986, Bednorz and $M\ddot{u}ller$ discovered the possibility of superconductivity at unusually high temperatures in a ceramic material consisting of four elements; Lnthium, barium, copper, and oxygen.

These discoveries could result in large scale commercial applications for cheap and efficient electricity aslong as superconducting wires that operated above liquid nitrogen temperatures of 80K can be manufactured in vast quantities.

2.2 Proximity effect

Cuprate superconductors above the critical temperature T_c do not behave as conventional metals. Up to now superconuctivity has been rather conventional, however, a possible exception that has grasped the attention of theoretical physicists is the giant proximity effect. Many physcists have observed that in Josephson junctions, with high temperature superconducting electrondes supercurrent can penetrate the interface of the normal conducting material to a thickness of $1000 - 10000\left(\times 10^{-10}\right)$m. However, such reports have been met with some reservations due to the conflict with current theory, and also due to exprimental problems. The conventional theory was published by De Gennes, not long after Meissner discovered that a superconductor and a normal conductor affect each other when brought in contact. De Gennes proposed that cooper pairs drift from the superconducting material to the normal metal several times, over a certain distance ξ. This distance whas later named the coherence length in the normal conducting material.

2.3 The magnetic Vector potential

The electric field is sometimes defined as the gradient of a scalar potential function; $\mathbf{E} = -\nabla V$. There is no scalar potential for magneticic field $\mathbf{B}$, however it can be shown as; $\mathbf{B} = \nabla \times \mathbf{A}$. $\mathbf{A}$ is known as the vector potential, but unlike the scalar potential is not associated with work directly. The magnetic vector potential relies on ampere's law and is equated in terms of either the current or current density.

$$\mathbf{A} = \frac{\mu_0 i}{4\pi} \oint \frac{d\mathbf{l}}{\mathbf{r}} \tag{1}$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} \oint \frac{j d\mathbf{S}\, d\mathbf{l}}{\mathbf{r}} \tag{2}$$

The vector potential is easier to calculate than the magnetic field from a given source current and is usually used when describing electromagnetic waves.

Since the curl of $\mathbf{A}$ gives the magnetic field, based on the vector identity $\nabla\times\nabla C = 0$, then any function where by the curl of the gradient is zero can be added to the vector potential, hence $\mathbf{A}' = \mathbf{A} + \nabla\phi$. This is named the Guage transformation.

2.4 Ginzburg-Landau Theory

The Ginzburg-Landau Theory is a mathematical theory that is used to model superconductivity. The theory only examines the macroscopic properties of a superconductor with help from general thermodynamics. Landau had previously established the theory of second-order phase transitions, however they both argued that the free energy of a superconductor at the interface of a superconducting transition, can be described using the complex order parameter Ψ, which is used to describe the depth at which the superconducting phase the system occurs.

The free energy density F_s can also be expressed in the form of the superconducting state in terms of the superfluid wavefunction $\Psi(r)$ which is complex. Near the critical temperature, the free energy density is thus;

$$F_s = F_n + \alpha |\Psi|^2 + \frac{1}{2}\beta |\Psi|^4 + \gamma \left| \nabla\Psi + \frac{2e\mathbf{A}}{-i\hbar}\Psi \right|^2 + \frac{1}{2\mu_0} (\mathbf{B} - \mathbf{B_E})^2 \tag{3}$$

Where F_n is the free energy in the normal phase, α, β are observable parameters, A is the electromagnetic vector potential and $(\mathbf{B} - \mathbf{B_E})$ is the magnetic field.

The free energy F_s depends on Ψ, however there is no assumptions for the microscopic explaination of Ψ. If we assume that a uniform superconductor is placed in a zero field, F_s can be expanded in powers of $|\Psi|^2$ and only include the first 2 terms.

2.4.1 Ginzburg-Landau 1^{st} equation

Taking the equation of the free energy density integrated over all space with respect to $\Delta\psi$ and $\Delta\beta$, we can achieve the two Ginzburg-Landau equations. The first of which is;

$$\frac{1}{2m} (-i\hbar\nabla + 2e\mathbf{A})^2 \Psi + \left(\alpha + \beta |\Psi|^2\right) \Psi = 0 \tag{4}$$

The first of the Ginzburg-Landau equation as shown in equation (4) is similar to the time-independent Schrödinger equation, which determines Ψ based on the magnetic field that is applied.

2.4.2 The derivation of current density J_s

The second Ginzburg-Landau equation is an expression for the Supperconducting current density J_s, which can be derived by considering the electron drift velocity.

$$\hat{J}(\mathbf{r}) = e\hat{\mathbf{v}}\hat{n} \tag{5}$$

Where,

$$\hat{\mathbf{v}} = \frac{\hat{p}}{m} = -i\hbar\frac{\nabla}{m} \tag{6}$$

$$\hat{n}(r) = \delta(\mathbf{r} - \mathbf{r_e}) \tag{7}$$

subbing equations 6 and 7 into 5 gives;

$$\hat{J}(\mathbf{r}) = \frac{-ei\hbar}{m}\nabla\delta(\mathbf{r} - \mathbf{r_e}) \tag{8}$$

This gives the supercurrent operator, when applied to the wavefunction and integrated over all space with respect to r_e will give the superconducting current $j(r)$. Hence,

$$\mathbf{j}(\mathbf{r}) = \int_{-\infty}^{\infty} \psi^*(\mathbf{r_e})\left(\frac{-i\hbar e}{m}\nabla\delta(\mathbf{r} - \mathbf{r_e})\right)\psi(\mathbf{r_e})\,d\mathbf{r_e} \tag{9}$$

However for the one dimensional case this equation can be rewritten;

$$\mathbf{j}(\mathbf{x}) = \frac{-i\hbar e}{m}\int_{-\infty}^{\infty} \psi(\mathbf{x})^*\left[\frac{\partial}{\partial x}\delta(\mathbf{x} - \mathbf{x_e})\right]\psi(\mathbf{x})\,dx \tag{10}$$

The problem with this equation resides with the fact that the delta function is not Hermition, hence it needs to be changed so that it is. Using the following identity it is possible turn the delta function into a hermition operator.

$$\frac{\partial}{\partial x}\delta(\mathbf{x} - \mathbf{x_e}) \rightarrow \frac{1}{2}\left[\frac{\partial}{\partial x}\delta(\mathbf{x} - \mathbf{x_e}) + \delta(\mathbf{x} - \mathbf{x_e})\frac{\partial}{\partial x}\right] \tag{11}$$

The integral of a delta function can be evaluated using a mathematical property of the delta function;

$$\int_{-\infty}^{\infty} f(x)\,\delta(x - x_e)\,dx = f(a) \tag{12}$$

Hence

$$-\int_{-\infty}^{\infty} \delta(\mathbf{x} - \mathbf{x_e})\,\psi(x)\,\frac{\partial}{\partial x}dx = -\psi(\mathbf{x})\,\frac{\partial}{\partial x}\psi(\mathbf{x})^* \tag{13}$$

Therefore

$$\mathbf{j}(\mathbf{x}) = \frac{-i\hbar e}{2m}\left[\psi(\mathbf{x})^*\,\frac{\partial}{\partial \mathbf{x}}\psi(\mathbf{x}) - \psi(\mathbf{x})\,\frac{\partial}{\partial x}\psi(\mathbf{x})^*\right] \tag{14}$$

2.5 The London Penetration depth

In 1935 F and H London modified an equation that was essential for electrodynamics, they obtained without changing maxwells equations, the Meissner effect. This lead Gorter and Casmir to develop the two fluid model. This model seperates the the electron system into its superconducting component with an electron density of n_s, and a normal component with an electron density n_n. Hence they achieved the total electron density, $n_0 = n_s + n_n$, and assumed that it behaved such that $n_s \to 0$ as $T \to 0$ and also $n_n = n_0$ if $T > T_c$. They derived from their phenomenological model, the London penetration depth λ_L.

$$\frac{1}{\lambda_L^2} = \frac{4\pi e^2 n_s}{m^* c^2} \tag{15}$$

Where m^* is the effective mass of the superconducting carriers.

λ_L arises from the failure of the Meissner effect, which occurs at the surface of a superconductor. The magnetic field can only penetrate the bulk of the superconducting material by an amount given by λ_L.

3 Modelling the tunneling effect using the finite potential wall example

When a superconducting material is in contact with a normal conducting material, the normal material carries some of the supercurrent for a finite depth into the material. This phenomena is known as the tunneling effect, or the proximity effect. It is possible to model what happens at the surface of the normal conducting material, by considering a paricle in a Quantum mechanical problem known as the finite potential wall. One would predict that the particle with a particular wavefunction, would be reflected at the surface of the wall due to the the potential. However, there is some penetration as if the particle is trying to tunnel through the surface of the normal conducting material.

The wave vector k is given by;

$$k = \sqrt{\frac{2mE}{\hbar^2}} \tag{16}$$

The model is split into two regions, we have region 1 and region 2, each one has different events occuring. In region 1 the incident particle travels towards the wall and is reflected in the opposite direction. We can denote the particle having a wavefunction as described by $\psi(x)_1$;

$$\psi(x)_1 = Ae^{ik_1x} + Be^{-ik_1x} \tag{17}$$

Where Ae^{ik_1x} describes the particle in the positive direction in this case to the right, and Be^{-ik_1x} for the particle travelling in the negative direction (the reflected particle). We also have to consider the particle in region 2. The particle in this region is illustrated by $\psi(x)_2$.

$$\psi(x)_2 = Ce^{ik_2x} + De^{-ik_2x} \tag{18}$$

Since there is no wave traveling in the negative direction, we can therefore say that;

$$\psi(x)_2 = Ce^{ik_2x} \tag{19}$$

Where k_1 and k_2 are the wave vectors in the two regions.

The three constants A, B and C can be related if we take into consideration the bounary conditions that $\psi(x)$ and $\frac{d\psi(x)}{dx}$ are continuous where the potential is discontinuous where x=0.

Hence,

$$A + B = C \tag{20}$$

$$ik_1(A - B) = ik_2C \tag{21}$$

It is possible to obtain the relationships for B/A and C/A, by rearranging a solving using simultaneous equations to get;

$$\frac{B}{A} = \frac{k_2 - k_1}{k_1^2 - k_1 k_2} \tag{22}$$

$$\frac{C}{A} = \frac{2k_1}{k_1 - k_2} \tag{23}$$

3.1 Reflection and Transmission coefficients

The reflection coefficient R is defined as the ratio of the intensity of the reflected probability current density $\frac{\hbar k}{m} |B|^2$ to the incident probability density $\frac{\hbar k}{m} |A|^2$, which gives the following relationship;

$$R = \frac{|B|^2}{|A|^2} \tag{24}$$

The transmission coefficient T is Similarly defined as the ratio of the intensity of the transmitted probability current density to the incident probability current density

Therefore;

$$T = \frac{|C|^2}{|A|^2} \tag{25}$$

Thus using equations 22 and 23 and also equation 14 we can obtain both the transmission and the reflection coefficients;

$$R = \frac{(k_2 - k_1)^2}{(k_1^2 - k_1 k_2)^2} \tag{26}$$

$$T = \frac{4k_1^2}{\left(k_1 - k_2\right)^2} \tag{27}$$

where R+T=1.

4 Modelling the penetration using the Vortex in the charged Bose liquid model

4.1 Bose-Einstein Condensation

A Bose-Einstein condensate is a phase formed by bosons that are cooled to temperatures close to absolute zero. The first of such a condensate was produced by Eric Cornell and Carl Wieman in 1995 at the university of colorado. Cornell and Wieman used a gas of rubidium atoms cooled to 170 nonokelvins. It was observed that a large fraction of the atoms collapse into the lowest quantum state. These quantum effects became apparent on a macroscopic level.

The properties of Bose-Einstein Condensates are not completely understood, such as spontaneously flowing out of their containers. This is a consequence of quantum mechanics; sytems can only obtain energy in discrete steps. If the system is at a very low temperature, so that it is in the lowest energy state, it is not possible to reduce its energy, not even by friction. Thus the fluid overcomes gravity due to adhesion between the fluid and the container wall, therefore the fluid will take up the most benificial position, which is usually all around the container.

The phenomenon of the Bose-Einstein condensation was predicted by Satyendra Nath Bose and Albert Einstein in 1920. It was originally based on Bose's work on the statistical mechanics of photons, which was then generalised by Einstein. The results from Bose and Einstein is the idea of a Bose gas, which obeys Bose-Einstein statistics. This describes the distribution of identical particles with integer spin, namely bosons. Bosonic particles, including the photon aswell as atoms such as helium-4, can share quatum states with eachother. Einstein hypothesized that cooling bosonic atoms to very low temperatures condenses them into the lowest available quantum state, resulting in a new form of matter. Such a transition occurs below a critical temperature, which for a uniform three-dimensional gas compising of non-interacting particles with no internal degree of freedom can be described by the following equation:

$$T_c = \left(\frac{2n}{3\zeta}\right)^{\frac{2}{3}} \frac{h^2}{2\pi m k_B} \tag{28}$$

where;

n is the particle density,

m is the mass per boson,

ζ is the Riemann zeta function $\frac{3}{2}\zeta \approx 2.614$

4.2 Meissner-Ochsenfeld effect and the London Equations

When a superconducting cylinder is exposed to an increasing magnetic field to a finite value B a surface current is induced whose magnetic field is able to cancel the applied field in the interior by Lenz's law. Since there is no resistance in a superconductor, the surface current is constant if the applied field is kept constant, the superconductor becomes like a perfect diamagnet below the critical temperature T_c. Eddy currents are induced when the temperature is higher than the critical temperature, because the external magnetic field increases. However the eddy currents decay quickly due to the resistance and applied magnetic field penetreates into the cylinder. If the temperature decreases below the critical temperature, a surface current is created to release the magnetic field from the bulk of the superconducting cylinder. This effect is known as the Meissner-Ochsenfeld effect. The transition from superconducting to normal conducting material can be compared to different thermodynamic phases. The first theoretical explaination for the Meissner-Ochsenfeld effect was established by H. and F. London, they assumed that the supercurrent is carried by some conduction electrons in the metal without friction, which can be called super-electrons. Their motion in a electric field can be described by the following equation;

$$m\frac{\partial \mathbf{v}}{\partial t} = -e\mathbf{E} \tag{29}$$

as previously defined in equation (3) the current density is.

$$\mathbf{J_s} = -en_s\mathbf{v} \tag{30}$$

Where n_s is the super-electron density. By partially differentiating equation (30) with respect to time, the equation can be equated to (27) which when rearranged gives Londons first equation;

$$\frac{\partial \mathbf{J_s}}{\partial t} = -en_s\frac{\partial \mathbf{v}}{\partial t} \tag{31}$$

hence.

$$\frac{\partial \mathbf{J_s}}{\partial t} = \frac{n_s e^2 \mathbf{E}}{m} \tag{32}$$

Equation (32) is Londons first equation. The rearrangement of this equation leads to the maxwells equation of iduction $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$, if the curl of equation (32) is taken. Thus;

$$\frac{m}{n_s e^2} \nabla \times \frac{\partial \mathbf{J_s}}{\partial t} = \nabla \times \mathbf{E} \tag{33}$$

substituting Maxwells equation;

$$\frac{m}{n_s e^2} \nabla \times \frac{\partial \mathbf{J_s}}{\partial t} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{34}$$

$$\frac{\partial}{\partial t} \left(\frac{m}{n_s e^2} \nabla \times \mathbf{J_s} + \mathbf{B} \right) = 0 \tag{35}$$

H and F London therefore assumed that the brackets tended to zero, hence giving the second London equation.

$$\nabla \times \mathbf{J_s} = -\frac{m}{n_s e^2} \mathbf{B} \tag{36}$$

This equation only applies to superconductors. When equation (36) is combined with the fourth maxwells equation; $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$, the equation for the magnetic field in a superconductor can be achieved.

$$\Delta \mathbf{B} - \frac{\mu_0 n_s e^2}{m} \mathbf{B} = 0 \tag{37}$$

Where; $\frac{\mu_0 n_s e^2}{m} = \frac{1}{\lambda_L^2}$ which is the london penetration depth. Therefore the magnetic field can be rewritten;

$$\Delta \mathbf{B} - \frac{1}{\lambda_L^2}\mathbf{B} = 0 \tag{38}$$

4.3 Vortex in the charged Bose liquid

In 1958 M.R Schafroth showed that below the ideal bose-gas condensation temperature, a gas of charged bosons demonstrated the Meissner-Ochsenfeld effect. Further work was conducted by L.L Foldy, whilst working on the one-particle excitation spectrum of the Coulomb Bose gas of high density. Foldy used the Bogoliubov approach, working at zero temperature, leading to the elementary excitation of the system, that has plasma oscillation energy characteristics for small momenta.

Charged Bose liquids are of high accademic interest and have been for a long time, inspite of experimental understanding of the Bose-Einstein condensation. Trapped ultra-cooled atoms make it possible to create ultracold plasma, by using lasers to trap and cool neutral atoms to temperatures close to absolute zero. Ionisation occurs using another laser, which gives the outer electrons enough energy to escape the electrical attraction of the associated ion. There is also experimental evidence for bosons with a charge of 2e in high temperature superconductors, physicists have reported normal state gap properties and unusual upper critical fields. Charged boson physics is also anticipated in a lattice of superconducting dots, provided suitable parameters are chosen so that single-electron tunnelling to be concealed. This means that only cooper pairs can tunnel between the two regions, using Josephson tunnelling which is similar to the model described in section (3). The model as described in the previous section, does not describe a typical superconducting junction as in Josephson junctions, instead it models an n-s junction. In order to model the transition in terms of bosons, the coulomb repulsion has to be taken into consideration, or else the bosons would collapse into the lowest highly localised state.

These advancements have brought the interest in charged boson liquids as an elementary reference system. A noninteracting gas of charged bosons cannot condense at any fixed magnetic field due to the one-dimensional motion in the lowest landau band. Nethertheless below the upper critical field, interacting charged bosons, have the ability to condense given that the collisions they make eliminate the one-dimensional demonstration of the density of states. A vortex in a charge boson liquid has a charged center and an electric field inside, whilst having a magnetic field identical to the Abrikosov vortex.

The Abrikosov vortex comprises of a vortex of supercurrent in a type II superconductor. Supercurrent moves around the normal material core of the vortex, which has a size comparable to the coherence length in Ginzberg-Landau theory ξ. The supercurrent decays at a distance of about λ_L from the core, in type II superconductors, $\lambda_L > \xi$. The supercurrent in the core induces a magnetic field with the total flux equal to a single flux Φ_0, hence the Abrikosov vortex is sometimes called a fluxon. The distribution of the magnetic field for a vortex can can be determined using the equation;

$$\mathbf{B}\left(\mathbf{r}\right) = \frac{\mathbf{\Phi_0}}{2\pi\lambda_L} K_0\left(\frac{r}{\lambda_L}\right) \approx \sqrt{\frac{\lambda_L}{\mathbf{r}}} \exp\left(-\frac{\mathbf{r}}{\lambda_L}\right) \tag{39}$$

Where K_0 is the McDonald function. When r tends to zero, the magnetic field will diverge logarithmically, for real world problems the magnetic field becomes;

$$\mathbf{B}(0) \approx \frac{\mathbf{\Phi_0}}{2\pi\lambda_L^2} ln(k) \tag{40}$$

where $k = \frac{\lambda_L}{\xi}$ is the Ginzberg-Landau parameter. In type II superconductors $k > \sqrt{2}$.

The magnetic field penetrates the boundary in terms of the Abrikosov Vortices, each vortex carries a string of magnetic field with a flux of Φ_0. They are able to form a lattice of vortices with an average vortex density equal approxiamtely to the applied external magnetic field.

The following section of the report is a derivation of the one-dimensional macroscopic condensate wave-function, substituting the bogoliubov displacement transformation into the equation of motion.

4.4 The Macroscopic condensate wavefunction

With the Vortex in charged bose liquid model, the supercurrent density, that penetrates the interface between the normal conductor and the superconductor, is modelled as charged bosons, with a long range coulomb interaction between them. The Hamiltonian of the charged bosons on a homogenius background, with an external magnetic field and a vector potential $\mathbf{A}(\mathbf{r})$ can be shown by the following equation.

$$\begin{aligned} H &= \int d\mathbf{r}\, \psi^*(\mathbf{r}) \left[-\frac{(\hbar\nabla - ie\mathbf{A}/c)^2}{2m} - \mu \right] \psi(\mathbf{r}) \\ &+ \frac{1}{2} \int d\mathbf{r} \int d\mathbf{r}\prime V(\mathbf{r} - \mathbf{r}\prime) \left[\psi^*(\mathbf{r}) \psi^*(\mathbf{r}\prime) \psi(\mathbf{r}\prime) \psi(\mathbf{r}) - 2n\psi^*(\mathbf{r}) \psi(\mathbf{r}) \right] \end{aligned} \tag{41}$$

where m, e, n, μ are the mass, charge, average density and the chemical potential of bosons, respectively. $V(\mathbf{r})$ is the coulomb potential. Using this hamiltonian, the equation of motion for the Heisenberg field operator $\psi(\mathbf{r}, t)$ can be derived. Assuming the density n to be very high, so that the dimensionless coulomb potential is small (repulsion is small), the occupation of a uniparticle state can be considered similar to the ideal bose gas. One of the states stays macroscopically occupied at absolute zero.

The expectation of the equation of motion $< \psi(\mathbf{r}, t) > \approx \sqrt{n}$ in a homogeneous system at T=0. Hence by substituting the Bogoliubov displacement transformation into the equation of motion, and collecting all c-number terms of ψ_0, the macroscopic condensate wavefunction can be derived from the following equation.

$$\left[\frac{(\hbar\nabla - ie\mathbf{A}/c)^2}{2m} + \mu\right]\psi(\mathbf{r}) = \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}')\left[\psi_0^*(\mathbf{r}')\psi_0(\mathbf{r}') - n\right]\psi_0(\mathbf{r}) \quad (42)$$

This equation, is different to the convensional Ginzberg-Landau and the Gross-Pitavskii equations, that includes the order parameters in the Bardeen Cooper and SchriefferCS and neutral superfluids. For this investigation, there is no external field and no chemical potential by convention, hence $\mathbf{A} = 0$ and $\mu = 0$. Therefore equation (40) can be rewritten as;

$$\frac{\hbar^2}{2m}\nabla^2\psi_0(\mathbf{r}) = \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}')\left[\psi_0^*(\mathbf{r}')\psi_0(\mathbf{r}') - n\right]\psi_0(\mathbf{r}) \quad (43)$$

Since $\psi_0(\mathbf{r})$ does not depend on **r'** it can be taken out of the integral. The equation can be simplified further by applying a potential operator on the right handside which will be denoted as $\Phi(\mathbf{r})$. Hence the equation becomes;

$$\frac{\hbar^2}{2m}\Delta\psi_0(\mathbf{r}) = \phi(\mathbf{r})\,\psi_0(\mathbf{r}) \tag{44}$$

Where $\phi(\mathbf{r}) = V(\mathbf{r} - \mathbf{r'}()\,1 - n)$. The potential is the coulomb repulsion hence $V(\mathbf{r}) = \frac{e^2}{4\pi\epsilon_0 \mathbf{r}}$. However the coulomb potential is interms of the subtracton of two vectors, $\mathbf{r} - \mathbf{r'}$, hence with a simple subtraction of 2 vectors the coulomb potential changes to;

$$V(\mathbf{r} - \mathbf{r}') = \frac{e^2}{4\pi\epsilon_0}\frac{1}{|\mathbf{r} - \mathbf{r}'|} \tag{45}$$

Thus;

$$\phi(\mathbf{r}) = \frac{e^2}{4\pi\epsilon_0}\frac{1}{|\mathbf{r} - \mathbf{r}'|}(1 - n) \tag{46}$$

Applying the wavefunction to $\phi(\mathbf{r})$ will give an equation for ϕ;

$$\phi = \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}')\left[\psi_0^*(\mathbf{r}')\,\psi_0(\mathbf{r}') - n\right] \tag{47}$$

$$\phi = \int d\mathbf{r}' V(\mathbf{r} - \mathbf{r}')\left[|\psi_0(\mathbf{r}')|^2 - n\right] \tag{48}$$

Taking the laplacian of ϕ, the following is true;

$$\Delta\phi(\mathbf{r}) = \int d\mathbf{r}'\left(\Delta_r V(\mathbf{r} - \mathbf{r}')\right)\left[|\psi_0(\mathbf{r}')|^2 - n\right] \tag{49}$$

The coulomb repulsion becomes quite interesting in this equation, because; using equations from electromagnetic field theory; $-\nabla\mathbf{V}(\mathbf{r}) = \mathbf{E}$ which leads to

$\nabla\mathbf{E} = \rho = e\delta\left(\mathbf{r}\right)$ hence, $\Delta_r V\left(\mathbf{r}-\mathbf{r'}\right) = \Delta_{r-r'} V\left(\mathbf{r}-\mathbf{r'}\right) = e\delta\left(\mathbf{r}-\mathbf{r'}\right)$. Due to the fact that only the one-dimensional case is being considered, $\Delta V\left(\mathbf{r}-\mathbf{r'}\right)$ changes to $\Delta V\left(\mathbf{x}-\mathbf{x'}\right) = e\delta\left(\mathbf{x}-\mathbf{x'}\right)$. Therefore using the delta function identity; $\int \delta\left(x-a\right) f\left(x\right) dx = f\left(a\right)$. $\Delta\phi$ can be rewritten.

$$\Delta\phi\left(\mathbf{x}\right) = \int d\mathbf{x}'\delta\left(\mathbf{x}-\mathbf{x}'\right)\left[\left|\psi_0\left(\mathbf{x}'\right)\right|^2 - n\right] \tag{50}$$

Using the delta function equality;

$$\int \delta\left(\mathbf{x}-\mathbf{x}'\right) f\left(x\right) = f\left(\mathbf{x}'\right) \tag{51}$$

we get the equation;

$$\Delta\phi = -\left(\left|\psi_0\left(\mathbf{x}'\right)\right|^2 - n\right) \tag{52}$$

From equations (44) and (52) the following two equations can be derived;

$$\Delta\psi_0 = \phi\psi_0 \tag{53}$$

$$\Delta\phi = n - \left|\psi_0\right|^2 \tag{54}$$

Where $\psi_0 = nf$, hence $f = \frac{\psi_0}{n}$. Therefore we can achieve solutions for the macroscopic condensate wavefunction.

$$\frac{d^2 f}{dx^2} = \phi\left(x\right) f\left(x\right) \tag{55}$$

$$\frac{d^2}{dx^2}\phi = 1 - \left|f\left(x\right)\right|^2 \quad if\ x < 0 \tag{56}$$

$$\frac{d^2}{dx^2}\phi = -\left|f\left(x\right)\right|^2 \quad if\ x > 0 \tag{57}$$

These three equations portray the condensate wavefunctions in different regions the two last equations show what happens when $x < 0$ and when $x > 0$, which give a good representation for the density distribution of bosons in the superconducting material and in the normal conducting material. Unfortunately current mathematical abilities prohibit the solutions to these differential equations.

5 Summary

In summation for the Theoretical investigation of the proximity effects of High temperature superconductors. The brief understanding of works done by Ginzburg, Landau and London was achieved using available resources. The derivation of the Ginzburg-Landau superconducting current was obtained and the reduction of the free energy density was understood to give Ginzburg Landau's first equation. A brief comprehension of the significance of the London penetration depth was transcribed using literature.

A model for the understanding of the penetration depth was worked through, using the finite step potential Quantum mechanical problem, giving quantitative expressions for the Refelection and Transmission Coefficients in terms of the wave vector. These coefficients provide a way to understand what happens, when the wavefunction penetrates the potential barrier. This gives a crude model, which describes what is going on at the proximity between the superconducting material and the normal conducting material.

To deepen the understanding of what happens at the interface of the two materials, the Meissner-Ochsenfeld effect was explained. Also an understanding into the ideas of Bose-Einstein condensation was conducting to lead into the study of the Abrikosov vortex and the macroscopic condensate wavefunction, which represents the density distribution of bosons in the two media. Three equations were derived using an adaption to the Gross-Pitaevskii type equations and also the Bogoliubov transformation, these describe the macroscopic wavefuncton for the one-dimensional no external field case. However, the resulting equations, make it extremely difficult to get a more comprehensive answer. This is due to the complexity of the resulting non-linear differential equations. Hopefully further studies will be conducted to try and find quantitative answers for the differential equations I derived. Although the understanding of the proximity effect is increasing all the time, this is a problem for the future theoretical physicist.

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