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# Lecture 5

$\nabla \times \vec{E} = -j\omega\mu \vec{H}$

$\nabla \times \vec{E} = j\omega\epsilon \vec{E} + \vec{J}$

$(\nabla^2 + k^2)\vec{E} = 0$ Helmholtz Equation

$k^2 = \omega^2 \mu \epsilon$

$2.4\times10^9$

$\mu$

## (d) The Harmonic Plane Wave

var w/t $e^{+j\omega t}$

$\vec{\mathcal{E}}(\vec{r},t) = \vec{f}(\hat{n}\cdot\vec{r} - vt) = \text{Re}\{\text{phasor}\}$

some fn

$\vec{E}(\vec{r},t) = \vec{E}_0\, e^{-j\vec{k}\cdot\vec{r}} e^{+j\omega t}$

complex constant vector (amp & phase) that doesn't depend on $\vec{r}$ or $t$.

plane wv

$\vec{k}\cdot\vec{r}$ is const in a plane $\perp$ $\vec{k}$ = constant

Then, if $\vec{\nabla}\cdot\vec{E} = 0$

$(\nabla^2 + k^2)\vec{E} = 0$

$\therefore\ \vec{E}_1 = \vec{E}_{01}\, e^{-j\vec{k}\cdot\vec{r}}$

اثبتوها بالبيت

to get $\vec{H}$ : $\vec{\nabla}\times\vec{E}_1 = -j\vec{k}\times\vec{E}_1 = -j\omega\mu\vec{H}_1$

$\vec{H}_1 = \dfrac{\vec{k}}{\omega\mu}\times\vec{E}_1$

OR: $\nabla \cdot \vec{H} = 0$

$$\vec{H}_2 = \vec{H}_{02} e^{-j\vec{k}\cdot\vec{r}}$$

$$\nabla \times \vec{H}_2 = -j\vec{k} \times \vec{H}_2 = j\omega\epsilon\vec{E}_2$$

$$\vec{E}_2 = -\frac{\vec{k}}{\omega\epsilon} \times \vec{H}_2 = -\frac{\vec{\beta}}{\omega\epsilon} \times \vec{H}_{02} e^{-j\vec{\beta}\cdot\vec{r}}$$

For both cases:

$$= -\frac{\omega\sqrt{\epsilon}\sqrt{\mu}}{\omega\epsilon} \times \vec{H}_{02} e^{-j\vec{\beta}\cdot\vec{r}}$$

$\eta$

$$\vec{k}\cdot\vec{k} = \omega^2\mu\epsilon \qquad \beta = \omega\sqrt{\epsilon\mu}$$

In General, $\vec{k} = \vec{\beta} - j\vec{\alpha}$ , $\vec{\alpha}$ and $\vec{\beta}$ are real

$$\beta^2 - \alpha^2 = Re(\omega^2\mu\epsilon)$$

$$\vec{\alpha}\cdot\vec{\beta} = -Im\{\omega^2\mu\epsilon\}$$

$\vec{\alpha}$ & $\vec{\beta}$ not necessary in same direction.

if $\vec{\beta}$ constant, constant phase?

non-uniform plane wave.

على الـ plane بتاع الـ wave اللى هوّ الـ constant phase عليه

الـ amplitude مكنه يتغير عليه.

ليه أخدنا الـ ref هوّ الـ phase ؟ لأنه يتغير أسرع.

العمودى فى اتجاه الـ $\vec{\beta}$.

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If $\in, \mu$ are real, we have two cases :

(i) $\vec{\alpha} = 0$ $\quad \vec{E_2} = \vec{E_1}$ and $\perp$ to $\vec{H_2}, \vec{H_1}$ $\quad (\vec{H}) \perp \vec{\beta}$

uniform plane wave.

(ii) $\vec{\alpha} \neq 0$ , $\vec{\alpha} \perp \vec{\beta}$ $\quad \vec{E_1} \neq \vec{E_2}$ , $\vec{H_1} \neq \vec{H_2}$

$\vec{E_1} \not\perp \vec{k}$ or $\vec{\beta}$

$\therefore \vec{H}$ is $\perp \vec{k}$

$$\vec{H_1} = \frac{\vec{k}}{\omega\mu} \times \vec{E_1}$$

$$\vec{E} \neq \vec{E_2}$$

T~~E~~-wave $\quad$ TE-wave

non-uniform plane wave

The energy stored we have (average)

$$\tilde{U}_e = \frac{1}{T}\int_0^T \frac{\vec{\mathcal{E}}\cdot\vec{\mathcal{E}}}{2\epsilon}\, dt \quad ; \quad T = \frac{2\pi}{\omega} \leftarrow \text{might be complex}$$

$$= \frac{1}{2\epsilon T}\int_0^T \mathrm{Re}\{\vec{E_0}\, e^{-j\vec{k}\cdot\vec{r}}\, e^{+j\omega t}\} \cdot \mathrm{Re}\{\vec{E_0}\, e^{-j\vec{k}\cdot\vec{r}}\, e^{+j\omega t}\}\, dt$$

$\uparrow$ assume phase $= 0$

$$= \frac{1}{2\epsilon T}\int_0^T \vec{E_0}\cdot\vec{E_0}\, e^{-\vec{\alpha}\cdot\vec{r}} \cos^2(\vec{\beta}\cdot\vec{r} - \omega t)\, dt$$

$$= \frac{\epsilon}{4}\,(\vec{E_0}\cdot\vec{E}^*)$$