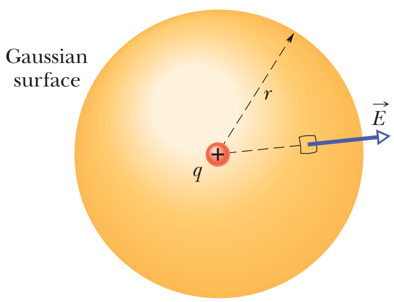


**Figure 23-9** A spherical Gaussian surface centered on a particle with charge $q$.

## Gauss' Law and Coulomb's Law

One of the situations in which we can apply Gauss' law is in finding the electric field of a charged particle. That field has spherical symmetry (the field depends on the distance $r$ from the particle but not the direction). So, to make use of that symmetry, we enclose the particle in a Gaussian sphere that is centered on the particle, as shown in Fig. 23-9 for a particle with positive charge $q$. Then the electric field has the same magnitude $E$ at any point on the sphere (all points are at the same distance $r$). That feature will simplify the integration.

The drill here is the same as previously. Pick a patch element on the surface and draw its area vector $d\vec{A}$ perpendicular to the patch and directed outward. From the symmetry of the situation, we know that the electric field $\vec{E}$ at the patch is also radially outward and thus at angle $\theta = 0$ with $d\vec{A}$. So, we rewrite Gauss' law as

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{\text{enc}}. \tag{23-8}$$

Here $q_{\text{enc}} = q$. Because the field magnitude $E$ is the same at every patch element, $E$ can be pulled outside the integral:

$$\varepsilon_0 E \oint dA = q. \tag{23-9}$$

The remaining integral is just an instruction to sum all the areas of the patch elements on the sphere, but we already know that the total area is $4\pi r^2$. Substituting this, we have

$$\varepsilon_0 E(4\pi r^2) = q$$

or

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \tag{23-10}$$

This is exactly Eq. 22-3, which we found using Coulomb's law.

### Checkpoint 3

There is a certain net flux $\Phi_i$ through a Gaussian sphere of radius $r$ enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to $r$, and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to $\Phi_i$?

### Sample Problem 23.03 Using Gauss' law to find the electric field

Figure 23-10$a$ shows, in cross section, a plastic, spherical shell with uniform charge $Q = -16e$ and radius $R = 10$ cm. A particle with charge $q = +5e$ is at the center. What is the electric field (magnitude and direction) at (a) point $P_1$ at radial distance $r_1 = 6.00$ cm and (b) point $P_2$ at radial distance $r_2 = 12.0$ cm?

**KEY IDEAS**

(1) Because the situation in Fig. 23-10$a$ has spherical symmetry, we can apply Gauss' law (Eq. 23-7) to find the electric field at a point if we use a Gaussian surface in the form of a sphere concentric with the particle and shell. (2) To find the electric field at a point, we put that point on a Gaussian surface (so that the $\vec{E}$ we want is the $\vec{E}$ in the dot product inside the integral in Gauss' law). (3) Gauss' law relates the net electric flux through a closed surface to the net enclosed charge. Any external charge is not included.

***Calculations:*** To find the field at point $P_1$, we construct a Gaussian sphere with $P_1$ on its surface and thus with a radius of $r_1$. Because the charge enclosed by the Gaussian sphere is positive, the electric flux through the surface must be positive and thus outward. So, the electric field $\vec{E}$ pierces the surface outward and, because of the spherical symmetry, must be *radially* outward, as drawn in Fig. 23-10$b$. That figure does not include the plastic shell because the shell is not enclosed by the Gaussian sphere.

Consider a patch element on the sphere at $P_1$. Its area vector $d\vec{A}$ is radially outward (it must always be outward from a Gaussian surface). Thus the angle $\theta$ between $\vec{E}$ and $d\vec{A}$ is zero. We can now rewrite the left side of Eq. 23-7 (Gauss' law) as

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E \cos 0\, dA = \varepsilon_0 \oint E\, dA = \varepsilon_0 E \oint dA,$$

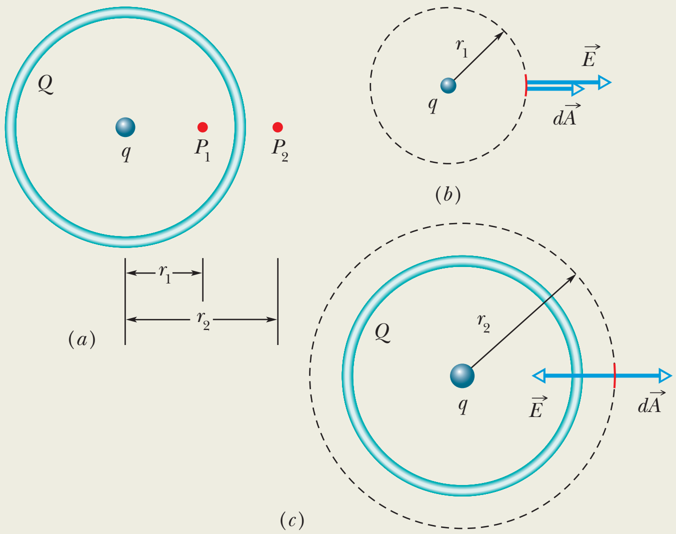


**Figure 23-10** (*a*) A charged plastic spherical shell encloses a charged particle. (*b*) To find the electric field at $P_1$, arrange for the point to be on a Gaussian sphere. The electric field pierces outward. The area vector for the patch element is outward. (*c*) $P_2$ is on a Gaussian sphere, $\vec{E}$ is inward, and $d\vec{A}$ is still outward.

where in the last step we pull the field magnitude $E$ out of the integral because it is the same at all points on the Gaussian sphere and thus is a constant. The remaining integral is simply an instruction for us to sum the areas of all the patch elements on the sphere, but we already know that the surface area of a sphere is $4\pi r^2$. Substituting these results, Eq. 23-7 for Gauss' law gives us

$$\varepsilon_0 E 4\pi r^2 = q_{\text{enc}}.$$

The only charge enclosed by the Gaussian surface through $P_1$ is that of the particle. Solving for $E$ and substituting $q_{\text{enc}} = 5e$ and $r = r_1 = 6.00 \times 10^{-2}$ m, we find that the magnitude of the electric field at $P_1$ is

$$E = \frac{q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{5(1.60 \times 10^{-19}\,\text{C})}{4\pi(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.0600\,\text{m})^2} = 2.00 \times 10^{-6}\,\text{N/C}. \quad \text{(Answer)}$$

To find the electric field at $P_2$, we follow the same procedure by constructing a Gaussian sphere with $P_2$ on its surface. This time, however, the net charge enclosed by the sphere is $q_{\text{enc}} = q + Q = 5e + (-16e) = -11e$. Because the net charge is negative, the electric field vectors on the sphere's surface pierce inward (Fig. 23-10*c*), the angle $\theta$ between $\vec{E}$ and $d\vec{A}$ is 180°, and the dot product is $E\,(\cos 180°)\,dA = -E\,dA$. Now solving Gauss' law for $E$ and substituting $r = r_2 = 12.00 \times 10^{-2}$ m and the new $q_{\text{enc}}$, we find

$$E = \frac{-q_{\text{enc}}}{4\pi\varepsilon_0 r^2} = \frac{-[-11(1.60 \times 10^{-19}\,\text{C})]}{4\pi(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.120\,\text{m})^2} = 1.10 \times 10^{-6}\,\text{N/C}. \quad \text{(Answer)}$$

Note how different the calculations would have been if we had put $P_1$ or $P_2$ on the surface of a Gaussian cube instead of mimicking the spherical symmetry with a Gaussian sphere. Then angle $\theta$ and magnitude $E$ would have varied considerably over the surface of the cube and evaluation of the integral in Gauss' law would have been difficult.

## Sample Problem 23.04 Using Gauss' law to find the enclosed charge

What is the net charge enclosed by the Gaussian cube of Sample Problem 23.02?

### KEY IDEA

The net charge enclosed by a (real or mathematical) closed surface is related to the total electric flux through the surface by Gauss' law as given by Eq. 23-6 ($\varepsilon_0\Phi = q_{\text{enc}}$).

***Flux:*** To use Eq. 23-6, we need to know the flux through all six faces of the cube. We already know the flux through the right face ($\Phi_r = 36$ N·m²/C), the left face ($\Phi_l = -12$ N·m²/C), and the top face ($\Phi_t = 16$ N·m²/C).

For the bottom face, our calculation is just like that for the top face *except* that the element area vector $d\vec{A}$ is now directed downward along the $y$ axis (recall, it must be *outward* from the Gaussian enclosure). Thus, we have $d\vec{A} = -dA\hat{\text{j}}$, and we find

$$\Phi_b = -16\,\text{N}\cdot\text{m}^2/\text{C}.$$

For the front face we have $d\vec{A} = dA\hat{\text{k}}$, and for the back face, $d\vec{A} = -dA\hat{\text{k}}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{\text{i}} + 4.0\hat{\text{j}}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)\,\text{N}\cdot\text{m}^2/\text{C} = 24\,\text{N}\cdot\text{m}^2/\text{C}.$$

***Enclosed charge:*** Next, we use Gauss' law to find the charge $q_{\text{enc}}$ enclosed by the cube:

$$q_{\text{enc}} = \varepsilon_0\Phi = (8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(24\,\text{N}\cdot\text{m}^2/\text{C}) = 2.1 \times 10^{-10}\,\text{C}. \quad \text{(Answer)}$$

Thus, the cube encloses a *net* positive charge.