$$
\frac{d}{dz}\begin{bmatrix} \widehat{u}_{\alpha,\beta} \\ \widehat{v}_{\alpha,\beta} \\ \widehat{w}_{\alpha,\beta} \\ \widehat{du}_{\alpha,\beta}/dz \\ \widehat{dv}_{\alpha,\beta}/dz \\ \widehat{dw}_{\alpha,\beta}/dz \end{bmatrix} = \frac{d\widehat{\mathbf{u}}_{\alpha,\beta}}{dz} = A_{\alpha,\beta}\widehat{\mathbf{u}}_{\alpha,\beta}, \tag{6}
$$

whose matrix is given by

$$
A_{\alpha,\beta} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{2\alpha^2(1-\sigma)+\beta^2(1-2\sigma)}{1-2\sigma} & \frac{\alpha\beta}{1-2\sigma} & 0 & 0 & 0 & \frac{-\mathrm{i}\alpha}{1-2\sigma} \\ \frac{\alpha\beta}{1-2\sigma} & \frac{2\beta^2(1-\sigma)+\alpha^2(1-2\sigma)}{1-2\sigma} & 0 & 0 & 0 & \frac{-\mathrm{i}\beta}{1-2\sigma} \\ 0 & 0 & \frac{(\alpha^2+\beta^2)(1-2\sigma)}{2(1-\sigma)} & \frac{-\mathrm{i}\alpha}{2(1-\sigma)} & \frac{-\mathrm{i}\beta}{2(1-\sigma)} & 0 \end{bmatrix}.
$$

This matrix has two eigenvalues, $\lambda = \pm k^2 = \pm(\alpha^2+\beta^2)$, with algebraic multiplicity equal to 3 and geometric multiplicity equal to 2. The matrix of eigenvectors is

$$
M_{\alpha,\beta} = \begin{bmatrix} \alpha^2 & \mathrm{i}\alpha & -\beta/k & \mathrm{i} & \mathrm{i}\alpha & \alpha\beta \\ \alpha\beta & \mathrm{i}\beta & \alpha/k & 0 & \mathrm{i}\beta & \beta^2 \\ -\mathrm{i}\alpha k & 4k\,(1-\sigma) & 0 & -\alpha/k & -4k\,(1-\sigma) & \mathrm{i}\beta k \\ \alpha^2 k & 0 & -\beta & -\mathrm{i}k & 0 & -\alpha\beta k \\ \alpha\beta k & 0 & \alpha & 0 & 0 & -k\beta^2 \\ -\mathrm{i}\alpha k^2 & k^2\,(3-4\sigma) & 0 & \alpha & k^2\,(3-4\sigma) & -\mathrm{i}\beta k^2 \end{bmatrix}
$$

and its associated Jordan form is

$$
J_{\alpha,\beta} = M_{\alpha,\beta}^{-1}A_{\alpha,\beta}M_{\alpha,\beta} = \begin{bmatrix} k & -\mathrm{i}k/\alpha & 0 & 0 & 0 & 0 \\ 0 & k & 0 & 0 & 0 & 0 \\ 0 & 0 & k & 0 & 0 & 0 \\ 0 & 0 & 0 & -k & 0 & 0 \\ 0 & 0 & 0 & 0 & -k & 0 \\ 0 & 0 & 0 & 0 & -\mathrm{i}k/\beta & k \end{bmatrix}.
$$

The solution of the differential equation (6) is

$$
\widehat{\mathbf{u}}_{\alpha,\beta}(z) = M_{\alpha,\beta}\exp(J_{\alpha,\beta}z)M_{\alpha,\beta}^{-1}\widehat{\mathbf{u}}_{\alpha,\beta}^{0}, \tag{7}
$$

where $\widehat{\mathbf{u}}_{\alpha,\beta}^{0}$ is the boundary value of $\widehat{\mathbf{u}}_{\alpha,\beta}$ at $z = 0$, which can be written as