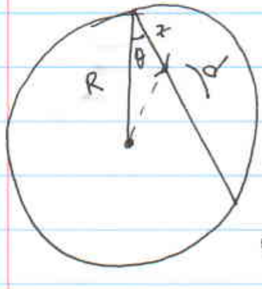
R = radius of Earth

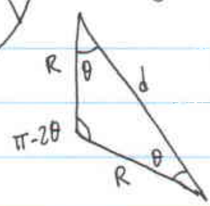
x = distance travelled along chord

d = length of chord

θ = angle between chord and radius

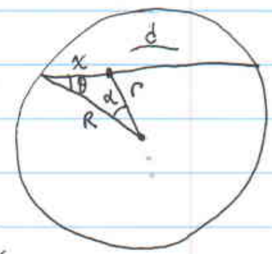


Solving for d using Law of Cosines



$$d^2 = R^2 + R^2 - 2R^2\cos(\pi - 2\theta)$$

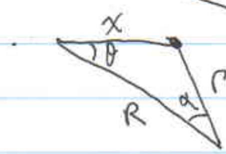
$$d = R\sqrt{2(1-\cos(\pi-2\theta))} \quad (1)$$



Rotating reference frame so chord is x-axis

~~r = distance to center after travelling distance x along chord~~

α = angle between effective ~~radius r and radius R~~

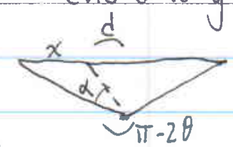


Law of Sines $\quad \dfrac{\sin\theta}{r} = \dfrac{\sin\alpha}{x} \quad (2)$

Solving for r

(angle α over total angle subtended by chord equals distance x over chord length)

$$\frac{\alpha}{\pi - 2\theta} = \frac{x}{d} \quad (3)$$



Two equations (2 and 3) with two unknowns (α and r) lets us solve for r in terms of x and θ

$r = \dfrac{x\sin\theta}{\sin\alpha}$ where $\alpha = \dfrac{x(\pi-2\theta)}{d}$ and $d = R\sqrt{2(1-\cos(\pi-2\theta))}$ by (1). (4)

Solving for acceleration as a function of x and θ

By Shell Theorem, $|\vec{a}| = \dfrac{GM}{r^2}$. $|\vec{a}|$ in the x-direction is given by $a_x = \dfrac{GM}{r^2}\cos(\theta + \alpha)$. Resubstituting,

$$a_x = \frac{GM\cos\left(\theta + \frac{x(\pi-2\theta)}{d}\right)}{\left(\frac{x\sin\theta}{\sin\left(\frac{x(\pi-2\theta)}{d}\right)}\right)^2}$$