

Magnetic Monopoles

We consider the relation

$$B^\alpha = \frac{1}{\sqrt{\gamma}} F_{\mu\nu} \text{-----} (1)$$

In flat spacetime,

$$B^\alpha = F^{\mu\nu} \text{-----} (2)$$

Again

$$B^\alpha = g^{\beta\mu} g^{\gamma\nu} F_{\mu\nu} \text{-----} (3)$$

Now if we transform from flat spacetime to curved spacetime, the left side of the last two equations transform to the left side of the first equation. Consequently the right sides of the last two should produce the right hand side of (1)

Considering the transformation from equation (3) to (1) we have

$$B'^\alpha = g'^{\beta\mu} g'^{\gamma\nu} F'_{\mu\nu} = F'^{\mu\nu}$$

Again from (1)

$$B'^\alpha = \frac{1}{\sqrt{\gamma'}} F'^{\mu\nu}$$

This inconsistency means that the quantity defined by equation (1) does not represent physically the transformed magnetic field.

But if we define:

$$B^\alpha = F^{\mu\nu}$$

Then,

1) There is no inconsistency as such.

2) We have $\nabla \mathbf{B} \neq 0$