

Lab 3, GEOL 202: Precipitation, Runoff and Channel Flows on Titan.

(adapted from Noah Finnegan, and based on [Perron et al., 2006](#))

Please **show all of your work** for the calculations below. **Pay attention to the bolded sections to make sure you complete everything that is asked for each question.**

Hint: Keep close track of your units when deriving all equations.

Titan is the largest moon of Saturn. The planet's surface is composed primarily of water ice. Its atmosphere is primarily nitrogen, but has clouds of methane that form, suggesting that there is also methane precipitation and hence methane "rain storms". Supporting the latter inference, large liquid methane lakes exist on Titan's surface, making it the only planet other than Earth where we know there to be stable bodies of liquid at the surface.



In late 2004, the Cassini Satellite launched a probe (known as the Huygens Probe) into orbit around Titan. In early 2005, as the Huygens Probe descended through Titan's atmosphere, it returned spectacular images of dendritic channel networks carved on Titan's surface. Although Titan's surface temperature is only about 94° K, it nevertheless appears to host active river processes. Because of the presence of methane lakes, it is widely believed that the river channels carved on Titan are due to flowing methane on the surface. Methane is the most abundant component of Titan's atmosphere after nitrogen, and at the temperature and pressures on Titan's surface, it should be stable in liquid form. From all appearances, Titan has an active methane cycle that in many ways is analogous to the water cycle on Earth. *However, almost nothing is known, in particular, about methane precipitation rates (i.e., methane rainfall rates) on Titan. Knowledge of methane precipitation rates would aid enormously in better understanding how methane cycles through Titan's atmosphere.*

In this week's lab we'll apply our sedimentology and sediment transport knowledge that we've gained in class to **estimate methane precipitation** rates on Titan from its sediment deposits. **NOTE: YOU MUST SHOW YOUR WORK TO GET FULL CREDIT FOR THIS LAB.** (There are 14 equations, each worth 2.38 pts to give 33.33 points total for this lab).

1. We'll assume that we can calculate precipitation based on the drainage area of a river as we did in lecture. We'll also assume evaporation is essentially zero (probably not a bad assumption for Titan given that there are large lakes of standing methane on its surface), and that there is zero transpiration (since there's no vegetation). **Write an expression that relates methane precipitation rate (P), drainage area (A), and discharge (Q_w):** $Q_w = A(P - ET)$

2. Assuming a rectangular channel geometry, **write an expression that relates channel width (W), depth (H), velocity (U), and discharge (Q_w):** $Q_w = uwh$

3. **Combine the equations you derived in steps 1 and 2 and solve for precipitation.** If you do this correctly, you'll see you've eliminated the variable Q_w. That's good, this will make it easier to solve for precipitation. : $U = A(P - ET) / WH$

4. It turns out, we can also eliminate the water velocity variable from the equation you derived in the previous step. We'll do this using the Manning Equation, which is a semi-empirical equation used to predict velocity in rivers. The Manning Equation states that the mean velocity of a flowing river (U) can be computed from the formula:

$$U = \frac{H^{2/3} S^{1/2}}{n}$$

where n is an empirical roughness coefficient. H is channel depth and S is channel slope. **Derive the units of n using dimensional analysis** (use L for units of length, T for units of time, and M for units of mass as we did in class). $n = (D^{1/6}) / (8.1 \sqrt{g})$

5. **Using the definition of U given above and substitute it into your equation for precipitation.** You should be left with an equation where precipitation is a function of H , W , S , A , and n .

6. So now if we only knew A , W , S , H , and n , we'd be able to calculate precipitation on Titan. Fortunately, these are all parameters we can measure or estimate! **Using Figure 2, please trace the boundary of the watershed upstream of the point A. You can do this directly on the image, with graph paper, by tracing it on a computer, or using any other means you'd like. Use the area you trace out to estimate the drainage area (A) in m^2 of the catchment upstream of point A.** Note that the light colored areas are higher elevation than darker regions. You'll need to use the scale bar in the image to figure out the drainage area. **Report the drainage in km^2 in the table at the end of the lab.**

7. Now let's measure the channel width. You can do this directly from the image using a ruler. Upstream of point A and downstream of the obvious tributary junction in the catchment (Figure 2), **estimate the channel width (W) in 5 locations.** Take the average of these measurements and report the **mean channel width in the table at the end of the questions.** You'll need to use a ruler and the scale bar to convert between the map scale and the real scale.

8. Let's try measuring slope. It turns out that the smart folks at NASA outfitted the Cassini satellite with a stereo imaging system. Consequently, we know that the elevation of the white-ish regions in the headwaters of our catchment are approximately 150 m higher than the point labeled A (Figure 2). **Using this information, estimate the channel slope (S) of the river and report it in the table at the end of the questions.** Remember that slope = rise/run, so you'll also need to figure out the approximate length of the river (the "run" in the rise/run formula). You can do this using Figure 2.

9. Ok, so now let's deal with flow depth (H). One way we can approach this is to ask what is the minimum flow depth that we can get that will allow us to move sediment in our river channel? Use the equation for critical Shields Stress, τ^*_c , to **derive an expression of the minimum channel depth, H , necessary to move a given grain size, D . Don't actually calculate values yet. Just derive the formula.**

$$\tau^*_c = \frac{\rho_f g H S}{(\rho_s - \rho_f) g D} = \sim 0.03 - 0.08$$

10. To make our calculations simpler, we'll assume $\tau^*_c = 0.045$ which is common for rivers with gravel beds. To calculate flow depth, we'll still need to know the size (D) and density of sediment (ρ_s) that is being moved by the river, as well as the density of the flow itself (ρ_f). We already know slope (S) from above. Happily, we know the density of the sediment being moved because it is water ice ($\rho_s = 900$

kg/m³). Additionally, we know the density of liquid methane on Titan's surface to be $\rho_f = 450 \text{ kg/m}^3$. Also happily for us, the Huygens probe took a picture of rounded, river transported cobbles in one of the outflow channels (Figure 3). **Measure the range of approximate grain diameter of 5 particles in the image, take the average of these and report your best guess at the average particle size in the table at the end of the questions.**

11. Now that we have an expression for the mean flow depth needed in our channel, **what are the maximum and minimum flow depths (H) given the range of possible τ^*_c values? Make sure to show your work.**

12. Ok - so now the last thing we need to do is figure out how to deal with the roughness coefficient, n . From empirical observations on Earth, we've discovered that the roughness coefficient scales with the grain size of sediment in the river following:

$$n = \frac{D^{1/6}}{8.1\sqrt{g}}$$

Calculate n , using the proper gravitational acceleration for Titan ($g = 1.45 \text{ m/s}^2$), you must report the units of n for full credit.

13. Sweet, you now have everything you need to calculate precipitation! Given the possible variations in H , **what's the range of possible precipitation rates on Titan?** Please report these rates in *cm/hr*. **Make sure to show your work.**

14. **How do these compare to typical rain rates on Earth?** Given this first-order estimation, is the methane cycle on Titan analogous (in terms of rates) to the water cycle on Earth? *Note: Global precipitation rates derived from NASA satellites range from 0.1 mm/hour to 50 mm/hour.*

Congratulations - you are ready to be a planetary scientist!

Report your variables measured from the image in this table (or create a similar table on the assignment you're turning in). You must report the units for full credit.

	Value	Unit
Drainage Area		
Average channel width		
Median grain size		
Channel slope		

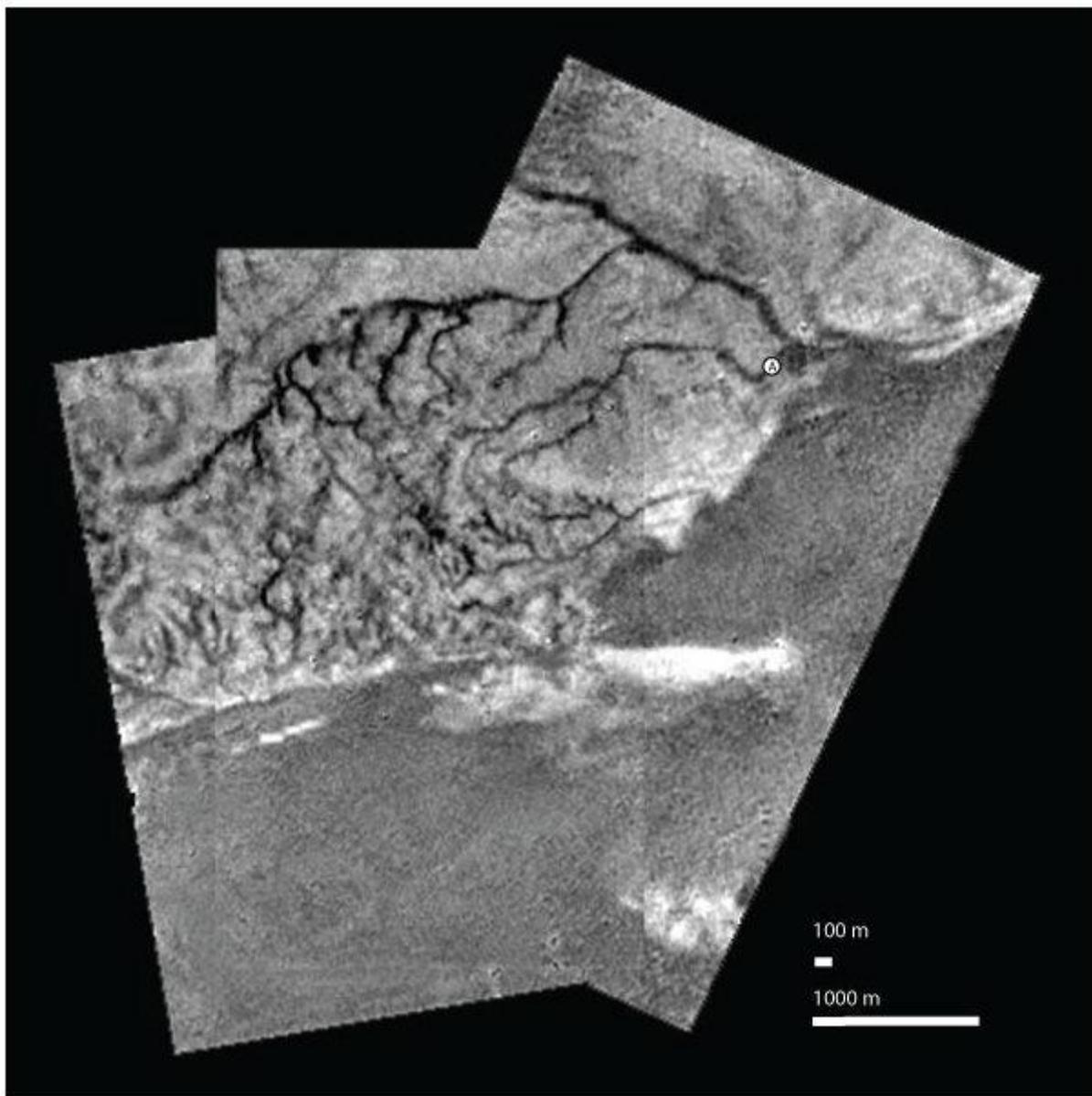


Figure 1: Drainage network on Titan's surface from Cassini-Huygens probe

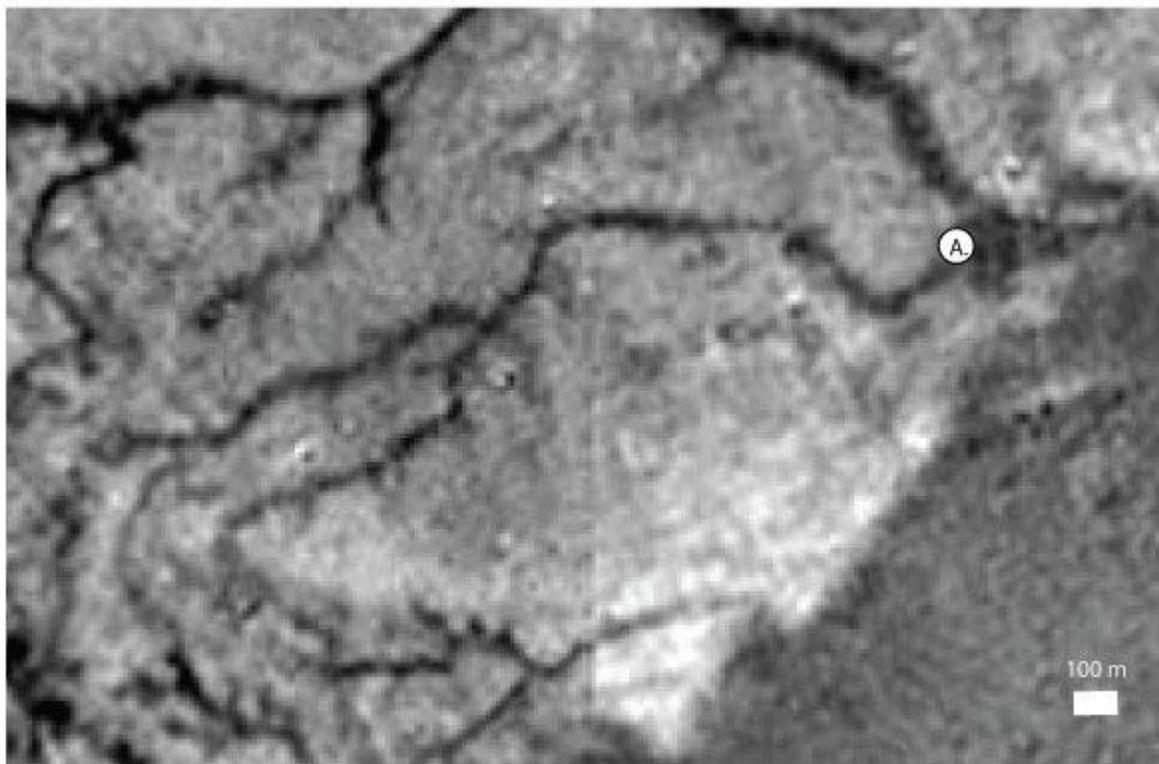


Figure 2: Blow-up of channel system shown in Figure 1. Note that the point A. is the same in both images

Table of Useful Variables

Variable	Value on Titan
ρ_s - sediment density	900 kg/m ³ - Water Ice
ρ_f - fluid density	450 kg/m ³ - Liquid Methane
g – gravitational acceleration on Titan	1.35 m/s ²
τ_c^* - Critical Shields Stress	0.03 - 0.08



Figure 3: Image of rounded ice cobbles taken by Huygens probe on Titan's surface. Note that the scale only applies to the rocks in the foreground (i.e. the lower 1/2 of the image).