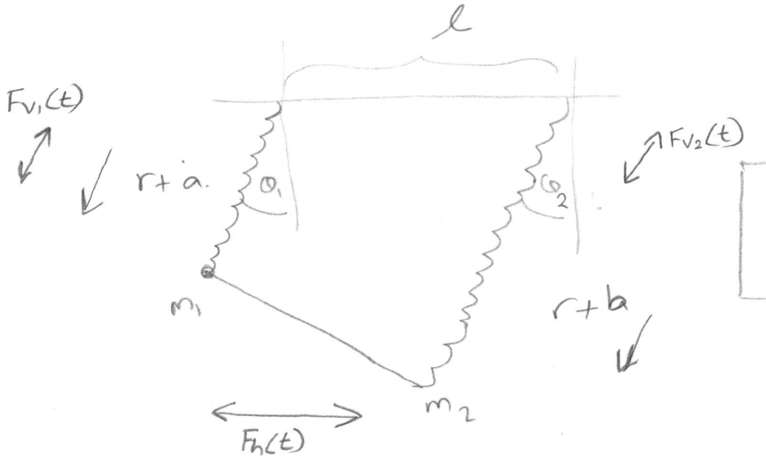


Spring stiffness = k
unstretched length = r

Damping

Spring $\Rightarrow$ $C_s\dot{a}$ and or $C_s\dot{b}$

Pendulum $\Rightarrow$ $C_p\dot{\theta}_1$ & $C_p\dot{\theta}_2$

$$T = \frac{1}{2}m_1\dot{a}^2 + \frac{1}{2}m_2\dot{b}^2 + \frac{1}{2}m_1(r+a)^2\dot{\theta}_1^2 + \frac{1}{2}m_2(r+b)^2\dot{\theta}_2^2$$

$$V = \frac{1}{2}ka^2 + \frac{1}{2}kb^2 - m_1 g(r+a)\cos\theta_1 - m_2 g(r+b)\cos\theta_2$$

b coordinate

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{b}}\right) = m_2 \ddot{b} \qquad \frac{\partial \mathcal{L}}{\partial b} = \frac{m_2 (2b+2r)\dot{\theta}_2^2}{2} - kb + g m_2 \cos\theta_2$$

$$\frac{\partial D}{\partial \dot{b}} = C_s \dot{b}$$

$$m_2 \ddot{b} + \overbrace{\frac{m_2 (2b+2r)\dot{\theta}_2^2}{2}}^{m_2 \dot{\theta}_2^2 (r+b)} + kb - g m_2 \cos\theta_2 + C_s \dot{b} = Q_b$$

$$m_2 \ddot{b} = Q_b - m_2 \dot{\theta}_2^2 (r+b) + kb - g m_2 \cos\theta_2 - C_s \dot{b}$$

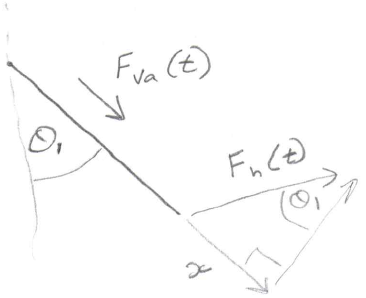
$$\ddot{b} = \frac{Q_b + m_2 \dot{\theta}_2^2 (r+b) - kb + g m_2 \cos\theta_2 - C_s \dot{b}}{m_2}$$

a. <u>coordinate</u>

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot a}\right) = m_1 \ddot a \qquad \frac{\partial \mathcal{L}}{\partial a} = \frac{m_1(2a+2r)\dot\theta_1^2}{2} - ka + gm_1\cos\theta_1$$

$$\frac{\partial D}{\partial \dot a} = C_s \dot a$$

$$m_1\ddot a - \overbrace{\frac{m_1(2a+2r)\dot\theta_1^2}{2}}^{m_1\dot\theta_1^2(a+r)} + ka - gm_1\cos\theta_1 + C_s\dot a = Q_a$$



$$\sin\theta_1 = \frac{x}{F_n(t)}$$

$$x = F_n(t)\sin\theta_1$$

$$(\delta_1+\delta_2)Q_x = F_n(t)\left[\sin\theta_1\hat i + \cos\theta_1\hat j\right]\cdot\delta_1 + F_v(t)\delta_2\uparrow$$

Similarly for b
* only 16,67Hz for b.

$$\Rightarrow F_n(t)\sin\theta + F_v(t)$$

↳ occurs twice @ parametric frequency & 16,67Hz

$$m_1\ddot a = Q_a - m_1\dot\theta_1^2(a+r) + ka - gm_1\cos\theta_1 - C_s\dot a$$

$$\ddot a = \frac{Q + m_1\dot\theta_1^2(a+r) - ka + gm_1\cos\theta_1 - C_s\dot a}{m_1}$$

θ_1 Coordinate

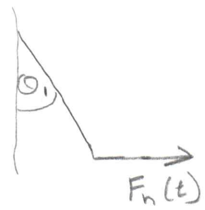
$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1} = m_1 \dot{\theta}_1 (r+a)^2$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) = m_1 \ddot{\theta}_1 (a+r)^2 + 2 m_1 \dot{\theta}_1 \dot{a} (r+a)$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = -g m_1 \sin\theta_1 (a+r)$$

$$\frac{\partial D}{\partial \dot{\theta}_1} = C_p \dot{\theta}_1$$

$$m_1 \ddot{\theta}_1 (r+a)^2 + 2 m_1 \dot{\theta}_1 \dot{a} (r+a) + g m_1 \sin\theta_1 (a+r) + C_p \dot{\theta}_1 = Q_{\theta_1}$$



$$\left.\begin{aligned} &\delta_{q_1} Q_{\theta_1} = F_n(t)\left[\sin\theta\,\hat{i} + \cos\,\hat{j}\right](a+r)\delta\theta_1 \\ &\Rightarrow (a+r) F_n(t) \cos\theta_1 \end{aligned}\right\} \text{Similarly for } \theta_2.$$

$$\Longrightarrow$$

$$m_1 \ddot{\theta}_1 (r+a)^2 = Q_a - 2 m_1 \dot{\theta}_1 \dot{a} (r+a) + g m_1 \sin\theta_1 (a+r) - C_p \dot{\theta}_1$$

$$\ddot{\theta}_1 = \frac{Q_{\theta_1} - 2 m_1 \dot{\theta}_1 \dot{a} (r+a) - g m_1 \sin\theta_1 (a+r) - C_p \dot{\theta}_1}{m_1 (r+a)^2}$$

θ_2 coordinate

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 \dot{\theta}_2 (r+b)^2 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 \ddot{\theta}_2 (r+b)^2 + 2 m_2 \dot{\theta}_2 \dot{b} (r+b)$$

$$\frac{\partial L}{\theta_2} = -g m_2 \sin \theta_2 (r+b)$$

$$\frac{\partial D}{\partial \dot{\theta}_2} = C_p \dot{\theta}_2$$

$$m_2 \ddot{\theta}_2 (r+b)^2 + 2 m_2 \dot{\theta}_2 \dot{b} (r+b) + g m_2 \sin \theta_2 (r+b) + C_p \dot{\theta}_2 = Q_{\theta_2}$$

$$m_2 \ddot{\theta}_2 (r+b)^2 = Q_{\theta_2} - 2 m_2 \dot{\theta}_2 \dot{b} (r+b) + g m_2 \sin \theta_2 (r+b) - C_p \dot{\theta}_2$$

$$\ddot{\theta}_2 = \frac{Q_{\theta_2} - 2 m_2 \dot{\theta}_2 \dot{b} (r+b) - g m_2 \sin \theta_2 (r+b) - C_p \dot{\theta}_2}{m_2 (r+b)^2}$$

where $(b+r)$ & $\dot{b}$ are replaced with the holonomic constraint

Holonomic Constraint (2)

$$\left[(r+a)\sin\theta_1 + l - (r+b)\sin\theta_2\right]^2 + \left[(r+a)\cos\theta_1 - (r+b)\cos\theta_2\right]^2 - l^2 = 0$$

* making θ_1 or θ_2 subject of formula yields imaginary numbers. Avoid

* Making a or b subject of formula gives ~~no~~ two roots with large formula. - difficult to work with

* making $(a+r)$ or $(b+r)$ subject of the formula gives a better equation to work with.

Holonomic constraint $(r+b) = B$ subject

Mathematica

$$\#1 = 2A\cos\theta_1 \cos\theta_2$$

$$\#2 = \pm\sqrt{2}\ \text{sqrt}\Big(\underbrace{-A^2+\ell^2}_{1} + \underbrace{A^2\cos(2\theta_1-2\theta_2)}_{2} - \underbrace{\ell^2\cos(2\theta_2)}_{3} - \underbrace{2A\ell\sin\theta_1}_{4} - \underbrace{2A\ell\sin(\theta_1-2\theta_2)}_{5}\Big)$$

$$\#3 = \underbrace{2\ell\sin\theta_2}_{6} + \underbrace{2A\sin\theta_1\sin\theta_2}_{7}$$

$$B = \frac{\#1+\#2+\#3}{2(\cos\theta_2^2+\sin\theta_2^2)} \Rightarrow \frac{1}{2}\big(\overbrace{\#1+\#2+\#3}^{x}\big) \ \& \ \frac{1}{2}(\#1-\#2+\#3)$$

$$B = (r+b)$$

$$B' = (r+b)' = b'$$

$$B = \frac{1}{2}\sqrt{x}$$

$$= \frac{1}{2}x^{+1/2}$$

$$\dot{B} = \frac{1}{4}x^{-1/2}\cdot\dot{x}$$

$$= \frac{\dot{x}}{4\sqrt{x}}$$

let $y = 1 + 2 + 3 + 4 + 5$

$$\#2 = -\sqrt{2}\sqrt{y}$$

$$= -\sqrt{2}\,y^{1/2}$$

$$\#\dot{2} = \frac{-\sqrt{2}}{2}\dot{y}\,y^{-1/2}$$

$$= \frac{-\sqrt{2}\dot{y}}{2\sqrt{y}}$$

$$2 = A^2\cos(2\theta_1-2\theta_2)$$

$$= A^2[\cos 2\theta_1\cos 2\theta_2 + \sin 2\theta_1\sin 2\theta_2]$$

$$5 = -2A\ell\sin(\theta_1-2\theta_2)$$

$$= -2A\ell[\sin\theta_1\cos 2\theta_2 - \cos\theta_1\sin 2\theta_2]$$

Time derivatives of holonomic constraint

$$\#\dot{1} = 2\dot{a}\cos\theta_1\cos\theta_2 - 2(a+r)\dot{\theta}_1\sin\theta_1\cos\theta_2 - 2(a+r)\dot{\theta}_2\cos\theta_1\sin\theta_2$$

$$\dot{6} = 2\ell\dot{\theta}_2\cos\theta_2$$

$$\dot{7} = 2\left[\dot{a}\sin\theta_1\sin\theta_2 + (a+r)\dot{\theta}_1\cos\theta_1\sin\theta_2 + (a+r)\sin\theta_1\dot{\theta}_2\cos\theta_2\right]$$

$$\#\dot{3} = \dot{6} + \dot{7}$$

$$\dot{3} = \ell^2 2\dot{\theta}_2\sin 2\theta_2$$

$$\dot{1} = -2\dot{a}(a+r)$$

$$\begin{aligned}\dot{2} = {} & 2\dot{a}(a+r)(\cos 2\theta_1\cos 2\theta_2) \\ & -(a+r)^2 2\dot{\theta}_1\sin 2\theta_1\cos 2\theta_2 \\ & -(a+r)^2\cos 2\theta_1\, 2\dot{\theta}_2\sin 2\theta_2 \\ & +2\dot{a}(a+r)(\sin 2\theta_1\sin 2\theta_2) \\ & (a+r)^2 2\dot{\theta}_1\cos 2\theta_1\sin 2\theta_2 \\ & (a+r)^2\sin 2\theta_1\cdot 2\dot{\theta}_2\cos 2\theta_2\end{aligned}$$

$$\dot{4} = -2\dot{a}\ell\sin\theta_1 - 2(a+r)\ell\dot{\theta}_1\cos\theta_1$$

$$\begin{aligned}\dot{5} = {} & -2\dot{a}\ell\sin\theta_1\cos 2\theta_2 \\ & -2(a+r)\ell\dot{\theta}_1\cos\theta_1\cos 2\theta_2 \\ & -2(a+r)\ell\sin\theta_1\cdot 2\dot{\theta}_1\sin 2\theta_2 \\ & +2\dot{a}\ell\cos\theta_1\sin 2\theta_2 \\ & -2(a+r)\ell\dot{\theta}_1\sin\theta_1\sin 2\theta_2 \\ & +2(a+r)\ell\cos\theta_1\cdot 2\dot{\theta}_2\cos 2\theta_2\end{aligned}$$