Biot Number

$$B_i = \frac{h r_o}{k} \Rightarrow \frac{1}{B_i} = \frac{k}{h r_o}$$

h = heat treatment coefficient
k = thermal conductivity
r_o = radius of cylinder

Fourier Number

$$\tau = \frac{\alpha t}{r_o^2}$$

α = thermal diffusivity
t = time
r_o = radius of cylinder

Thermal Diffusivity

$$\alpha = \frac{k}{\rho \cdot c_p}$$

k = thermal conductivity
ρ = density
C_p = specific heat capacity

Properties of 1095 steel

$k = 51.9 \frac{W}{m\text{-}°C}$
$\rho = 7870\ kg/m^3$
$C_p = 486\ J/kg\text{-}°C$
$h = 7.9\ W/m^2\text{-}°C$
$r_o = 10'' = .254\ m$
$t = 1\ hr = 3600\ sec$

* r_o is assuming largest crankshaft raw stock at 20" diameter

Refer to Heisler Chart, Fig 4-16 after finding $\frac{1}{B_i}$ an τ.

$$\frac{1}{B_i} = \frac{51.9}{(7.9)(.254)} = 25.9$$

$$\tau = \frac{\alpha t}{r_o^2} \Rightarrow \alpha = \frac{51.9}{(7870)(486)} = .0000136 = 1.36 \times 10^{-5}\ m^2/s$$

$$\tau = \frac{(1.36 \times 10^{-5})(3600)}{(.254)^2} = .76 \approx 0.8$$

Looking at chart a) centerline temperature in figure 4-16, the $\theta_o = \frac{T_o - T_\infty}{T_i - T_\infty}$ value is: .9

$$T_c = 15.6°C + .9(1260°C - 15.6°C) = 1135°C$$

$$T_c = T_a + \theta_o (T_i - T_a)$$

$$T_c = 60°F + .9(2300°F - 60°F) = 2076°F$$

T_c = centerline temperature
T_a = ambient temperature
T_i = initial surface temperature

$$\boxed{\text{After 1 hr, a 20" diameter forging has an internal centerline temperature of 2076°F}}$$

According to www.weatherbase.com, the average Yearly temp in Mount Vernon is 48.9°F (9.39°C)

$$T_c = 9.39°C + .9(1260°C - 9.39°C) = 1134°C$$

$$T_c = 48.9°F + .9(2300°F - 48.9°F) = 2074°F$$

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Core temp of forging must be at least 1250°F before going into normalizing furnace. 1200°F is a safer number due to uncentainty in thermocouples.

$$T_C = T_a + \theta_o (T_i - T_a)$$

1200°F ⇒ 649°C
cooling in average ambient air temp of 48.9°F ⇒ 9.39°C
2300°F ⇒ 1260°C

$$649°C = 9.39°C + \theta(\overset{1260°C}{\cancel{}} - 9.39°)$$

$$649 = 9.39 + \theta(1250.61)$$
$$639.61 = \theta(1250.61)$$
$$\boxed{\theta = .511}$$

$$\frac{1}{B_i} = \frac{51.9}{(7.9)(.254)} = 25.9$$

$$\tau = \frac{(1.36\times 10^{-5})(t)}{(.254)^2} = .0002108(t) = 10 \quad \therefore\ t = 47438\text{ sec} \Rightarrow \boxed{13.1\text{ hours}}$$

A 20" diameter crankshaft must cool around 13 hours after forging at 2300°F to get a core temp of 1200°F.

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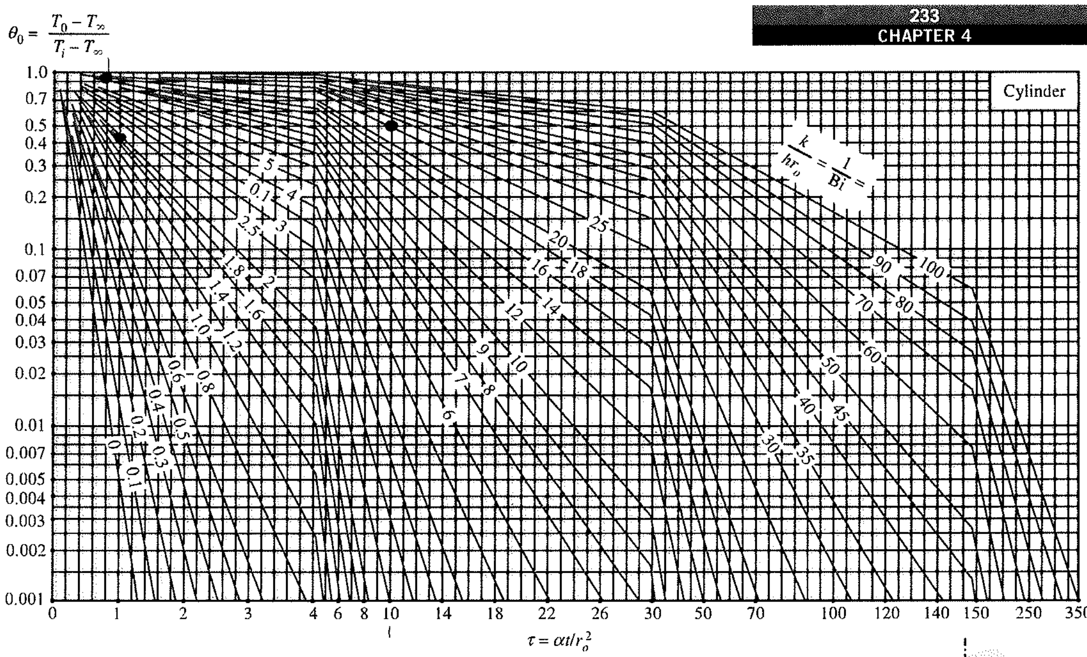
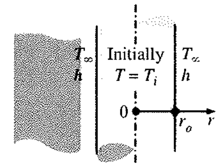
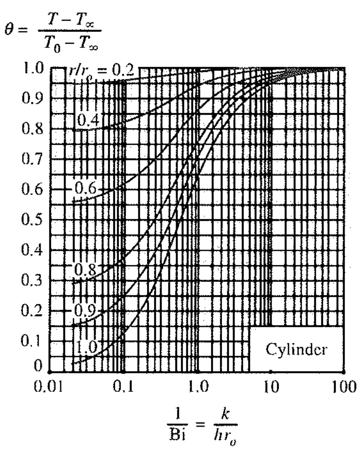
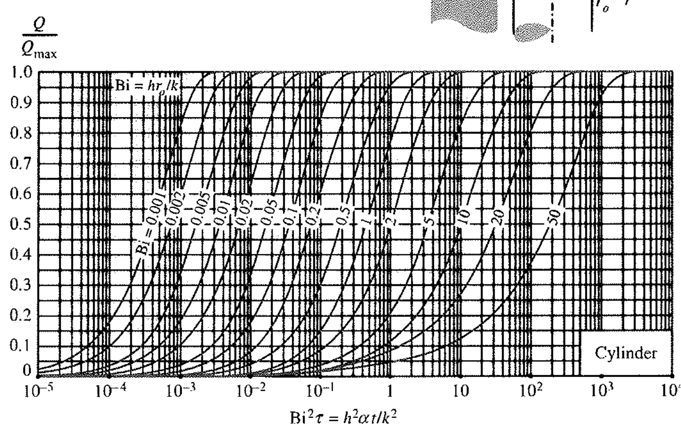


(*a*) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



(*c*) Heat transfer (from H. Gröber et al.)

FIGURE 4–16

Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

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It yields

$$T(t) = 279°\text{C}$$

which is practically identical to the result obtained above using the Heisler charts. Therefore, we can use lumped system analysis with confidence when the Biot number is sufficiently small.

EXAMPLE 4–5 **Cooling of a Long Stainless Steel Cylindrical Shaft**

A long 20-cm-diameter cylindrical shaft made of stainless steel 304 comes out of an oven at a uniform temperature of 600°C (Fig. 4–23). The shaft is then allowed to cool slowly in an environment chamber at 200°C with an average heat transfer coefficient of $h = 80\ \text{W/m}^2 \cdot °\text{C}$. Determine the temperature at the center of the shaft 45 min after the start of the cooling process. Also, determine the heat transfer per unit length of the shaft during this time period.

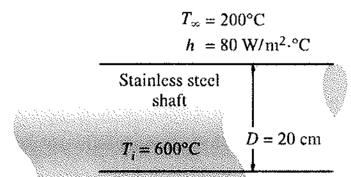

FIGURE 4–23
Schematic for Example 4–5.

SOLUTION A long cylindrical shaft is allowed to cool slowly. The center temperature and the heat transfer per unit length are to be determined.

Assumptions **1** Heat conduction in the shaft is one-dimensional since it is long and it has thermal symmetry about the centerline. **2** The thermal properties of the shaft and the heat transfer coefficient are constant. **3** The Fourier number is $\tau > 0.2$ so that the one-term approximate solutions are applicable.

Properties The properties of stainless steel 304 at room temperature are $k = 14.9\ \text{W/m} \cdot °\text{C}$, $\rho = 7900\ \text{kg/m}^3$, $c_p = 477\ \text{J/kg} \cdot °\text{C}$, and $\alpha = 3.95 \times 10^{-6}\ \text{m}^2/\text{s}$ (Table A–3). More accurate results can be obtained by using properties at average temperature.

Analysis The temperature within the shaft may vary with the radial distance r as well as time, and the temperature at a specified location at a given time can be determined from the Heisler charts. Noting that the radius of the shaft is $r_o = 0.1$ m, from Fig. 4–16 we have

$$\left.\begin{aligned} \frac{1}{\text{Bi}} &= \frac{k}{hr_o} = \frac{14.9\ \text{W/m} \cdot °\text{C}}{(80\ \text{W/m}^2 \cdot °\text{C})(0.1\ \text{m})} = 1.86 \\ \tau &= \frac{\alpha t}{r_o^2} = \frac{(3.95 \times 10^{-6}\ \text{m}^2/\text{s})(45 \times 60\ \text{s})}{(0.1\ \text{m})^2} = 1.07 \end{aligned}\right\} \quad \frac{T_0 - T_\infty}{T_i - T_\infty} = 0.40$$

and

$$T_0 = T_\infty + 0.4(T_i - T_\infty) = 200 + 0.4(600 - 200) = 360°\text{C}$$

Therefore, the center temperature of the shaft drops from 600°C to 360°C in 45 min.

To determine the actual heat transfer, we first need to calculate the maximum heat that can be transferred from the cylinder, which is the sensible energy of the cylinder relative to its environment. Taking $L = 1$ m,

$$m = \rho V = \rho \pi r_o^2 L = (7900\ \text{kg/m}^3)\pi(0.1\ \text{m})^2(1\ \text{m}) = 248.2\ \text{kg}$$
$$Q_{\max} = mc_p(T_\infty - T_i) = (248.2\ \text{kg})(0.477\ \text{kJ/kg} \cdot °\text{C})(600 - 200)°\text{C}$$
$$= 47{,}350\ \text{kJ}$$

$$\frac{1}{Bi} = \frac{k}{hr_o}$$

$$\tau = \frac{\alpha t}{r_o^2}$$

τ = Fourier Number

$Bi = \frac{hL}{k}$ ⟹ Biot Number

$\left[Bi = \frac{hr_o}{k}\right]$ ← same

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K of steel = $51.9\ \frac{W}{m\text{-}K}$ or $360\ \frac{BTU\text{-}in}{hr\text{-}ft^2\text{-}°F}$

$= 51.9\ \frac{W}{m°C}$