$$h(y) = \int 2y + 1 \, dy = y^2 + y + k$$

You'll note that we included the constant of integration, k, here. It will turn out however that this will end up getting absorbed into another constant so we can drop it in general.

So, we can now write down $\Psi(x, y)$.

$$\Psi(x, y) = x^2 y - 3x^3 + y^2 + y + k = y^2 + (x^2 + 1) y - 3x^3 + k$$

With the exception of the k this is identical to the function that we used in the first example. We can now go straight to the implicit solution using (4).

$$y^2 + (x^2 + 1) y - 3x^3 + k = c$$

We'll now take care of the k. Since both k and c are unknown constants all we need to do is subtract one from both sides and combine and we still have an unknown constant.

$$\begin{aligned} y^2 + (x^2 + 1) y - 3x^3 &= c - k \\ y^2 + (x^2 + 1) y - 3x^3 &= c \end{aligned}$$

Therefore, we'll not include the k in anymore problems.

This is where we left off in the first example. Let's now apply the initial condition to find c.

$$(-3)^2 + (0 + 1)(-3) - 3(0)^3 = c \qquad \Rightarrow \qquad c = 6$$

The implicit solution is then.

$$y^2 + (x^2 + 1) y - 3x^3 - 6 = 0$$

Now, as we saw in the separable differential equation section, this is quadratic in y and so we can solve for $y(x)$ by using the quadratic