

**Example 2** Solve the following IVP and find the interval of validity for the solution.

$$2xy - 9x^2 + (2y + x^2 + 1) \frac{dy}{dx} = 0, \quad y(0) = -3$$

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First identify  $M$  and  $N$  and check that the differential equation is exact.

$$\begin{aligned} M &= 2xy - 9x^2 & M_y &= 2x \\ N &= 2y + x^2 + 1 & N_x &= 2x \end{aligned}$$

So, the differential equation is exact according to the test. However, we already knew that as we have given you  $\Psi(x, y)$ . It's not a bad thing to verify it however and to run through the test at least once however.

Now, how do we actually find  $\Psi(x, y)$ ? Well recall that

$$\begin{aligned} \Psi_x &= M \\ \Psi_y &= N \end{aligned}$$

We can use either of these to get a start on finding  $\Psi(x, y)$  by integrating as follows.

$$\Psi = \int M dx \quad \text{OR} \quad \Psi = \int N dy$$

However, we will need to be careful as this won't give us the exact function that we need. Often it doesn't matter which one you choose to work with while in other problems one will be significantly easier than the other. In this case it doesn't matter which one we use as either will be just as easy.

So, we'll use the first one.